VOLUME LXIX

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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Yerkes Observatory
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MAY 1929

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ON THE ASTRONOMICAL IMPLICATIONS OF THE DE SITTER LINE ELEMENT FOR THE UNIVERSE

By RICHARD C. TOLMAN

ABSTRACT

Part I. Nature of De Sitter universe.—Using the form of line element proposed by Eddington, *exact expressions* are obtained *for the motion of particles and light-rays in the De Sitter universe*, which include the shape of the orbit, the velocity of motion in the orbit, the acceleration in the orbit, and, for purely radial motion, the integrated equations of motion.

Part II. Relation to astronomical measurements.—The relation is discussed between the co-ordinate r and the determinations of radial distance that would result from direct measurement with meter sticks, from measurements of parallax, from those of the apparent luminosity of distant objects, and from those of the apparent extension of distant objects. A relation between the proper density of distribution of objects and co-ordinate density is obtained. *Exact expressions* are derived for the *Doppler effect* in terms of *distances* and *velocities* or in terms of distances and orbital parameters, both for the case that the *observer* is at the *origin* and for the case that the *source* is at the origin of co-ordinates. The Doppler effect at perihelion, and the range of distance and range of time during which an approaching source would produce the reversed positive Doppler effect, are discussed.

Part III. Relation to astronomical findings.—The known facts as to the distances and Doppler effects for the extra-galactic nebulae are discussed on the assumption that these objects can be regarded as free particles in a De Sitter universe. Four possible hypotheses that would reconcile the presence of nebulae within the range of observation with the scattering tendency which exists in the De Sitter universe are presented and discussed. The most natural of these hypotheses is that of continuous entry in accordance with which nebulae are continually entering as well as leaving the range of observation. This hypothesis leads to a Doppler effect which tends to be positive and to increase with distance, but an overwhelming predominance of positive Doppler effects and a linear increase with distance could only be secured by placing additional conditions on the parameters for the orbits which do not appear inevitable. The other hypotheses appear less natural. They can be made to agree with the experimental facts by the introduction of *ad hoc* assumptions. *The conclusion is drawn that the De Sitter line element does not afford a simple and unmistakably evident explanation of our present knowledge of the distribution, distances, and Doppler effects for the extra-galactic nebulae.*

The correlation between distance and apparent radial velocity for the extra-galactic nebulae obtained by Hubble,[1] and the recent measurement of the Doppler effect for a very distant nebula made by Humason[2] at the Mount Wilson Observatory, make it desirable to consider once more the theoretical relations between distance and Doppler effect which could be expected from the form of line element for the universe proposed by De Sitter.

Consideration has already been given to these relations by De Sitter,[3] Weyl,[4] Eddington,[5] and others, and in particular by Silberstein,[6] who obtained conclusions not previously appreciated. The present article will give a systematic presentation of derivations from the De Sitter line element of interest for the problem at hand; the conclusions will be obtained in the form which seems most suitable for astronomical applications and their relation to methods of astronomical measurement shown; a number of new conclusions will be presented; and the possibility of correspondence between present astronomical findings and the De Sitter assumption will be discussed.

PART I. NATURE OF THE DE SITTER UNIVERSE

1. *The line element for the De Sitter universe.*—In accordance with the general theory of relativity, the nature of any space-time manifold is completely characterized by the expression for the line element ds^2 in terms of the co-ordinates x_1, x_2, x_3, and x_4 which are being used. For the space-time manifold of the universe as a whole, neglecting local deviations due to the neighborhood of stars or stellar systems, two essentially different forms of line element have been proposed due to De Sitter and to Einstein himself.

Both of these forms of line element may be considered as obtained, on the basis of differing assumptions, from Einstein's generalized equation

$$-8\pi T_{\mu\nu}=G_{\mu\nu}-\tfrac{1}{2}Gg_{\mu\nu}+\lambda g_{\mu\nu}\,, \qquad (1)$$

[1] *Proceedings of the National Academy*, **15**, 168, 1929.

[2] *Ibid.*, **15**, 167, 1929.

[3] *Monthly Notices, Royal Academy of Sciences*, **76, 77, 78**, 1916–1917.

[4] *Physik-Zeitschr.*, **24**, 230, 1923.

[5] *The Mathematical Theory of Relativity*, Cambridge, 1923.

[6] *The Theory of Relativity* (2d ed.), Macmillan, 1924.

which connects the material-energy tensor $T_{\mu\nu}$ with the contracted Riemann-Christoffel tensor $G_{\mu\nu}$ and the fundamental tensor $g_{\mu\nu}$. If we regard the universe on the whole as substantially empty and hence put the energy tensor equal to zero, and regard λ as a constant having some definite value, equation (1) leads to the De Sitter line element. Per contra, if we regard the universe as filled with matter of constant density and negligible velocity, and regard λ as a parameter which is dependent in a particular way on the total mass in the universe, equation (1) leads to the line element for the Einstein universe.

The essential difference in the character of the two universes is partly due to the difference in the assumptions as to the absence or presence of matter in the universe, but is also partly due to the difference in the assumptions as to the nature of the quantity λ. If the De Sitter point of view were correct, and we should be able to make the actual amount of matter in the universe smaller and smaller, the universe would approach more and more perfectly the form demanded by De Sitter with dimensions determined by the constant λ. On the other hand, if the Einstein point of view were correct, and we should gradually decrease the amount of matter in the universe, it would get smaller and smaller, due to the above-mentioned relation between total mass and the parameter λ, and finally in the complete absence of matter the universe would shrink to zero volume.

The expression for the De Sitter line element obtained from equation (1) in the manner indicated above can be written in a variety of forms depending on the choice of co-ordinate system. The mathematical nature of the line element can be most readily apprehended if co-ordinates are chosen so that it assumes the form

$$ds^2 = -R^2[d\omega^2 + \sin^2\omega\{d\zeta^2 + \sin^2\zeta(d\theta^2 + \sin^2\theta d\phi^2)\}], \qquad (2)$$

which will be recognized as an expression in angular variables for the line element on the surface of a four-dimensional sphere of radius R.

By a substitution of variables in accordance with the equations

$$\left.\begin{aligned} \cos\omega &= \cos\chi\cos it, \\ \cot\zeta &= \cot\chi\sin it, \end{aligned}\right\} \qquad (3)$$

(2) can be transformed into

$$ds^2 = -R^2 d\chi^2 - R^2 \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) + R^2 \cos^2 \chi dt^2 , \qquad (4)$$

which is a form often used for physical applications. By a further substitution of variables in accordance with the equation

$$\sin \chi = r/R , \qquad (5)$$

the line element can be written in the form

$$ds^2 = -\frac{1}{1 - r^2/R^2} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + (1 - r^2/R^2) dt^2 , \qquad (6)$$

which is the one that we shall use as the starting-point for the considerations of this article.[1]

By working out the values of the contracted Riemann-Christoffel tensor, all three of the foregoing forms for the line element can be shown to agree with equation (1), with the left-hand side equal to zero, provided we identify the constant λ by the relation $\lambda = 3/R^2$.

Other forms of the De Sitter line element are sometimes used, but we shall find equation (6) the most suitable for our purposes, since the variables occurring in it will later appear to be the most natural ones to correlate with the results of astronomical measurements.

We note at present that for all values of r from the origin $r = 0$ out to $r = R$, the co-ordinate $t = x_4$ is timelike in nature since g_{44} is always positive, while the other co-ordinates are spacelike in nature. Furthermore, since in the neighborhood of the origin, $r \ll R$, the line element reduces to the familiar form

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + dt^2 \qquad (6a)$$

applicable in flat space-time, we shall find it convenient to speak of r, θ, and ϕ as polar co-ordinates, regarding r as the radius from the origin and θ and ϕ as the polar and equatorial angles.

2. *The equations for a geodesic.*—The motion of particles and light in the space-time given by the line element (6) will be determined by the consideration that the corresponding world-lines will be geodesics. Fortunately, equation (6) is of the familiar form

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^{\nu} dt^2 \qquad (7)$$

[1] See Eddington, *loc. cit.*

with

$$e^{-\lambda}=e^{\nu}=(1-r^2/R^2)\,, \tag{8}$$

for which the forms taken by the geodesic equations

$$\frac{d^2x_\alpha}{ds^2}+\{\mu\nu,\ \alpha\}\frac{dx_\mu}{ds}\frac{dx_\nu}{ds}=0 \tag{9}$$

are well known.[1]

With $\alpha=2$ the geodesic equation is of such a form that a particle originally moving in the plane $\theta=\pi/2$, with $d\theta/ds=0$, will continue to move permanently in that plane in accordance with the equations

$$\frac{d^2\theta}{ds^2}=0\,,\qquad \frac{d\theta}{ds}=0\,,\qquad \theta=\frac{\pi}{2}\,, \tag{10}$$

and under this condition the remaining equations with $\alpha=1,\ 3,\ 4$ reduce to

$$\frac{d^2r}{ds^2}+\frac{1}{2}\frac{d\lambda}{dr}\left(\frac{dr}{ds}\right)^2-re^{-\lambda}\left(\frac{d\phi}{ds}\right)^2+\tfrac{1}{2}e^{\nu-\lambda}\frac{d\nu}{dr}\left(\frac{dt}{ds}\right)^2=0\,, \tag{11}$$

$$\frac{d^2\phi}{ds^2}+\frac{2}{r}\frac{dr}{ds}\frac{d\phi}{ds}=0\,, \tag{12}$$

$$\frac{d^2t}{ds^2}+\frac{d\nu}{dr}\frac{dr}{ds}\frac{dt}{ds}=0\,. \tag{13}$$

3. *Integration of the geodesic equations.*—The last two of these equations can be easily integrated to give

$$\frac{d\phi}{ds}=\frac{h}{r^2} \tag{14}$$

and

$$\frac{dt}{ds}=ke^{-\nu}=\frac{k}{1-r^2/R^2}\,, \tag{15}$$

where h and k are constants of integration. Evidently h is a parameter which can assume positive or negative values according to the direction of motion of the particle, but it should be specially noted that k is necessarily a positive parameter, if we are going to interpret t as the time and exclude the possibility that the proper time s and co-ordinate time t could ever run backward.

[1] See *ibid.*

For the special case of light, the parameters h and k both assume the value infinity, since for the propagation of light we have the fundamental requirement that the interval ds shall be equal to zero.

By substituting the values given by (14) and (15) into equation (11), it too can now be integrated. It is simpler, however, to substitute (14) and (15) into the original equation (6) for the line element. Doing so, and noting equations (10), we easily obtain

$$\frac{dr}{ds} = \pm\sqrt{k^2 - 1 + \frac{r^2}{R^2} - \frac{h^2}{r^2} + \frac{h^2}{R^2}}, \tag{16}$$

and by differentiation it can be shown that this is in fact an integral of equation (11). We have thus obtained the first integrals for all of the geodesic equations.

4. *The shape of orbit.*—We can now easily acquire information concerning the shape of the orbits taken by particles and light-rays in the De Sitter universe, by combining equations (16) and (14). Dividing one by the other and slightly rearranging the terms we obtain

$$d\phi = \frac{h\,dr}{r^2\sqrt{\frac{r^2}{R^2} + \left(k^2 - 1 + \frac{h^2}{R^2}\right) - \frac{h^2}{r^2}}} \tag{17}$$

The right-hand side of this equation can readily be integrated to give a complete expression for the shape of the orbits taken by particles in the De Sitter universe. A quicker idea of the nature of these orbits can be obtained, however, from the fact that the form of relation given by (17) is well known in Newtonian mechanics[1] as applying to the shape of orbit taken by a particle with a central repulsive force proportional to the radius r. Hence in the De Sitter universe the orbits of free particles, plotted in the polar co-ordinates r, θ, and ϕ, are in general curved away from the origin as though the particles were repelled by it.

The condition for perihelion or nearest approach of the particle to the origin will be given by the fact that $dr/d\phi$ will then obviously become zero. Hence perihelion occurs at a distance from the origin

[1] See, e.g., Boltzmann, *Vorlesungen über die Prinzipe der Mechanik*, Teil I, § 20, equation (40).

such that the quantity under the radical sign in equation (17) is equal to zero, giving us the following equation,

$$k^2-1+\frac{r_m^2}{R^2}-\frac{h^2}{r_m^2}+\frac{h^2}{R^2}=0\,, \tag{18}$$

connecting the parameters of the orbit with r_m, the minimum distance to which the particle approaches the origin.

For purely radial motion, i.e., with $h=0$, the value of $dr/d\phi$ will be indeterminate at perihelion, if it takes place at all, and the condition given by equation (18) reduces to

$$\frac{r_m}{R}=\sqrt{1-k^2}\,, \tag{19}$$

perihelion only occurring when the parameter k is less than unity, the particle passing through the origin for larger values of k.

For the special case of light-rays, the parameters h and k occurring in equation (17) assume the value infinity as already noted in §3.

Hence for the case of a light-quantum the orbital equation (17) takes the form

$$d\phi=\frac{dr}{r^2\sqrt{\frac{k^2-1}{h^2}+\frac{1}{R^2}-\frac{1}{r^2}}}\,, \tag{20}$$

which corresponds in Newtonian mechanics to an orbit in which the central force has become zero. Thus the paths of light-rays, plotted in the co-ordinates r, θ, ϕ, are straight lines. This can also easily be shown by the integration of equation (20), which leads to $r\cos\phi+ar\sin\phi=b$, where a and b are constants.

5. *The velocity of motion in the orbit.*—Although the shapes of the orbits of free particles plotted in our co-ordinates are the same as in Newtonian mechanics, with a central repulsive force proportional to the distance, the velocity of motion in the orbit is different. In terms of the proper time s, of the particle itself, the velocity of motion has already been given by equations (14) and (16), and by combining these with (15) we easily obtain for the components of velocity, in terms of the co-ordinate time t, the expressions

$$\frac{d\phi}{dt}=(1-r^2/R^2)\,\frac{h}{kr^2} \tag{21}$$

and

$$\frac{dr}{dt}=\pm\frac{(1-r^2/R^2)}{k}\sqrt{k^2-1+\frac{r^2}{R^2}-\frac{h^2}{r^2}+\frac{h^2}{R^2}}\,. \tag{22}$$

By introducing the condition for perihelion, given by (18), the latter of these equations can be re-written in the form

$$\frac{dr}{dt}=\pm\frac{(1-r^2/R^2)^2}{k}\sqrt{\frac{r^2}{R^2}-\frac{h^2}{r^2}-\frac{r_m^2}{R^2}+\frac{h^2}{r_m^2}}\,, \tag{23}$$

which is sometimes useful since it correlates the velocity with the distance of the particle away from perihelion.

For the special case of light, the parameters h and k occurring in (21) and (22) assume, as already noted, the value infinity, so that equations (21) and (22) become indeterminate. We can easily obtain expressions for the velocity of light, however, by making immediate use of the fundamental requirement that the interval ds is equal to zero for the motion of light. Returning to our original equation (6) for the line element and putting $ds^2=0$, we obtain in general for the velocity of light

$$\left(\frac{dr}{dt}\right)^2+(1-r^2/R^2)\left\{r^2\left(\frac{d\theta}{dt}\right)^2+r^2\sin^2\theta\left(\frac{d\phi}{dt}\right)^2\right\}=(1-r^2/R^2)^2\,. \tag{24}$$

For a purely radial motion of light this becomes

$$\frac{dr}{dt}=\pm(1-r^2/R^2)\,. \tag{25}$$

It will be immediately noted that all the foregoing expressions for velocity become zero at $r=R$. Hence all movement ceases at the radius R from the origin. Furthermore, since the integral of equation (25) shows that light would take an infinite time to travel over the radius R, we can never have any information of events happening at R or beyond and hence may speak of a horizon to the universe at this distance. It should be remarked, however, that this horizon is in a certain sense illusory, since another observer, located at a different origin, will also locate his horizon differently.

Returning to equation (23), it will be noted that the radial velocity of a particle becomes zero also at the perihelion $r=r_m$, as

indeed it must since this gives an alternative requirement for perihelion.

For the very special case of purely radial motion, $h=0$, the velocity of the particle becomes zero at the perihelion $r_m=R\sqrt{1-k^2}$ provided k is less than unity. But for values $h=0$ and k greater than unity it is evident from equation (22) that the velocity becomes zero only at the horizon $r=R$, and that a particle originally approaching the origin will gradually slow down, pass through the origin with a minimum velocity, and then speed up again as it continues away from the origin. Finally, for the intermediate case $h=0$ and $k=1$, equation (22) gives zero velocity for a particle at the origin $r=0$ itself.

6. *Acceleration in the orbit.*—Expressions for the angular and radial accelerations in the orbits can be arrived at by differentiating (21) and (22) with respect to the time. We thus obtain after some rearrangement of terms,

$$\frac{d^2\phi}{dt^2}=-\frac{2h}{kr^3}\frac{dr}{dt} \tag{26}$$

and

$$\frac{d^2r}{dt^2}=-\frac{2r/R^2}{1-r^2/R^2}\left(\frac{dr}{dt}\right)^2+\left(\frac{1-r^2/R^2}{k}\right)^2\left(\frac{r}{R^2}+\frac{h^2}{r^3}\right). \tag{27}$$

In accordance with equation (27), the radial acceleration is always positive when the velocity is zero and r lies between 0 and R. Hence a free particle which once reaches perihelion and starts to move away from the origin will never again return.

Putting the right-hand side of equation (27) equal to zero, we evidently obtain the condition for a maximum or minimum of velocity. As an interesting illustration of the behavior of particles in the De Sitter universe, we thus find, for example, for the simple case $h=0$ and $k=1$, that a free particle would have its maximum velocity well out toward the horizon at $r=R/\sqrt{3}$, where it would be moving with the enormous radial speed $2/3\sqrt{3}$, the radial speed of light itself being only $2/3$ at that distance from the origin.

It will also be noted from equation (27) that for the special case of pure radial motion, i.e., with $h=0$, the acceleration becomes zero at the origin $r=0$. Hence a free particle with $h=0$ and $k=1$, which

as we have seen makes dr/dt zero at the origin, would remain permanently at rest at the origin, and indeed the origin itself can be regarded as located on such a free particle. This is important since it means that we can consider ourselves as at the origin of such a system of co-ordinates.

7. *The integrated equations of motion for the orbit.*—Equation (22) for the radial velocity can be re-written in a standard form for integration, so that we can obtain an integrated equation connecting t and r. Furthermore, since the equation (17), connecting ϕ and r, can also be integrated, the equations of motion are susceptible of a complete solution in terms of elementary functions.

The resulting expressions, however, are rather complicated, and it will be sufficient for our purposes to restrict ourselves to the case of purely radial motions, with $h=0$. Doing so we can re-write equation (22) in the form

$$dt=\frac{kdr}{(1-r^2/R^2)\sqrt{k^2-1+r^2/R^2}}, \tag{28}$$

and this can be integrated to give us

$$t_2-t_1=\frac{R}{2}\log\frac{\sqrt{k^2-1+r^2/R^2}+kr/R}{\sqrt{k^2-1+r^2/R^2}-kr/R}\Bigg|_{r_1}^{r_2}, \tag{29}$$

or substituting the condition for perihelion given by (19), this can be re-written in the form

$$t_2-t_1=\frac{R}{2}\log\frac{\sqrt{\frac{r^2}{r_m^2}-1}+\sqrt{\frac{r^2}{r_m^2}\left(1-\frac{r_m^2}{R^2}\right)}}{\sqrt{\frac{r^2}{r_m^2}-1}-\sqrt{\frac{r^2}{r_m^2}\left(1-\frac{r_m^2}{R^2}\right)}}\Bigg|_{r_1}^{r_2}, \tag{30}$$

which applies to the motion of particles for which $h=0$ and $k<1$.

For the purely radial motion of light, equation (25) can be re-written in the form

$$dt=\frac{dr}{1-r^2/R^2}, \tag{31}$$

which has the integral

$$t_2-t_1=\frac{R}{2}\log\frac{R+r}{R-r}\Bigg|_{r_1}^{r_2}. \tag{32}$$

As stated above, the result shows that it would take an infinite length of time for light to travel to or from the horizon.

PART II. RELATION TO ASTRONOMICAL MEASUREMENTS

8. *The measurement of distances.*—The quantities r, θ, ϕ, and t occurring in our equation (6) for the line element can in the first instance be regarded only as co-ordinates corresponding to some particular possible method of partitioning space and time. An investigation must be made to obtain the correspondence between these co-ordinates and the results we should obtain by experimental methods which might conceivably be used for measuring distances and times.

We have already seen in §1 that r, θ, and ϕ are spacelike co-ordinates and t a timelike co-ordinate for all values of r less than the distance R to the horizon, and hence for all parts of the universe that can come under the investigation of an observer located even for an infinite length of time at the origin. Furthermore, in the immediate neighborhood of the origin, at distances near enough so that $r \ll R$, the line element degenerates into the simple form (6*a*), familiar in flat space-time, so that for nearby measurements r, θ, and ϕ become ordinary polar co-ordinates, agreeing with our customary measurements of distance and angle in Euclidean space.

As we go away from the origin, however, we find from the form of the line element, equation (6), that a rigid measuring-stick of invariant length, say ds_0, when held respectively parallel or perpendicular to the radius r, would give the co-ordinate readings $dr = \sqrt{1 - r^2/R^2}\, ds_0$, $r d\theta = ds_0$ and $r \sin\theta d\phi = ds_0$ instead of $dr = ds_0$, $r d\theta = ds_0$ and $r \sin\theta d\phi = ds_0$, as would be true with ordinary polar co-ordinates in a flat space. We thus recognize that the three-dimensional space, with which we are concerned, is not Euclidean flat space but is curved. And we note in particular that the co-ordinate r does not agree with measures of astronomical distance which would be obtained by laying rigid scales end to end from the origin to the object under observation.

Astronomical determinations of radial distance are, however, not made by such direct methods, but in general are either obtained from measurements of parallax or estimated by comparing the apparent

luminosity of the distant object with the absolute luminosity to be expected for objects of that class, or, in the case of objects such as the nebulae which have sufficient size, can be estimated by comparing the apparent extension of the object with the absolute extension to be expected. For correlation with these methods of determining distance, the co-ordinate r is particularly well adapted.[1]

First as to measurements of parallaxes or parallactic motions, the main assumption underlying their interpretation is that light travels in straight lines. And this as we saw in §4 is fulfilled if we regard r, θ, and ϕ as ordinary polar co-ordinates. Hence the parallactic determination of radial distance, which is basic for other astronomical methods, will give distances which agree with the co-ordinate r, even though direct measurement of the distances with rigid scales would not agree with r.

As to the estimation of radial distances from measurements of apparent luminosity, our co-ordinate system also offers advantages, since the propagation of light in straight lines assures the inverse-square law for the falling off of the intensity of the emitted light, just as is ordinarily assumed. Strictly, however, two theoretical corrections must still be applied to make such estimates of distance correspond to the co-ordinate r.

The first of these corrections arises from the difference between the proper time s for the object emitting the radiation and the co-ordinate time t which is used by the observer located at the origin of co-ordinates. The relation between these two quantities is given by equation (15),

$$\frac{dt}{ds}=\frac{k}{1-r^2/R^2}.$$

Hence, for example, if dN/dt as estimated from our observations is the rate at which quanta, arising from a particular kind of atomic transition, are leaving the luminous object,

$$\frac{dN}{ds}=\frac{dt}{ds}\frac{dN}{dt}=\frac{k}{1-r^2/R^2}\frac{dN}{dt} \tag{33}$$

[1] This is one of the main reasons for the choice of this co-ordinate system, which has hitherto been used to any extent only by Eddington.

will be the rate of emission of these quanta as measured by a local observer stationed on the object itself. And it is values of dN/ds rather than dN/dt that should be the same for different objects of the same class. Nevertheless, for the present, the correction indicated by equation (33) can apparently be neglected, since, as we shall later see, the quantity $k/(1-r^2/R^2)$ could probably differ from unity by not more than a few per cent for the objects so far investigated.

The second possible correction arises from the fact of the large Doppler effect present in the radiation from the nebulae. This means that the quanta associated with a given atomic transition arrive on a photographic plate, placed at the origin, with a wave-length which differs from that of similar quanta coming from a source that shows no Doppler effect. Hence, since the photographic plate is not equally sensitive to quanta of all wave-lengths, and the atmospheric cut-off is different for quanta of different wave-lengths, a second correction is theoretically required. This correction too can, however, apparently be neglected until we get far greater Doppler shifts than have so far been observed.

Finally, for estimates of distance depending on the apparent extension of distant objects, our co-ordinate system is also well adapted, since the propagation of light in straight lines combined with the form of the line element makes the quantities $r\,d\theta$ and $r\sin\theta d\phi$ correct expressions for the extension of a distant object as it would be measured by a local observer.

It will also be noted that proper motions receive a simple interpretation if we use the chosen co-ordinate system.

9. *The determination of densities of distribution.*—In addition to measurements of distances, the astronomer is interested in determinations of the densities of distribution in space of objects of different classes. To interpret such determinations with the help of our co-ordinate system, we may start from the fundamental relation

$$dV_0 ds = \sqrt{-g}\, dx_1\, dx_2\, dx_3\, dx_4\,, \tag{34}$$

where dV_0 is the proper volume and ds the proper time for a local observer. In terms of our co-ordinates this gives us

$$dV_0 ds = r^2 \sin\theta\, dr\, d\theta\, d\phi\, dt\,, \tag{35}$$

or writing for the element of co-ordinate volume

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi \,, \tag{36}$$

and substituting for dt/ds from equation (15), we have

$$dV_0 = \frac{k}{1 - r^2/R^2} \, dV \,, \tag{37}$$

where the parameter k refers to the motion of a free particle upon which the local observer is thought of as located. If the local observer is at rest in our system of co-ordinates, equation (37) reduces to

$$dV_0 = \frac{1}{\sqrt{1 - r^2/R^2}} \, dV \,. \tag{38}$$

If now we denote by ρ and ρ_0 the number of objects per unit volume as determined respectively in our system of co-ordinates and by the local observer, we may evidently write the equality

$$\rho dV = \rho_0 dV_0 \,, \tag{39}$$

since the actual number of objects in a given region must be an invariant. Hence substituting equations (37) or (38), we may write in general

$$\rho = \frac{k}{1 - r^2/R^2} \, \rho_0 \,, \tag{40}$$

or for the special case that the local observer is at rest,

$$\rho = \frac{1}{\sqrt{1 - r^2/R^2}} \, \rho_0 \,. \tag{41}$$

Since k can in no case be less than $\sqrt{1 - r^2/R^2}$, we see that the ratio ρ/ρ_0 necessarily increases as we go toward the horizon, for all possible ways of picking out the local observer.

10. *The Doppler shift* (*observer at the origin*).—In addition to measurements of distance and proper motion, the astronomer makes measurements of the Doppler effect in order to determine the radial velocity of motion of distant objects. We must now investigate the Doppler effect to be expected from free particles in the De Sitter universe. This can be done in two ways, taking either the observer or the emitting source as at the origin of co-ordinates.

Let us first take the observer as at the origin of co-ordinates, and let the source be at the distance r and moving with the radial speed dr/dt at the time of emission. If ds is the period of the emitting atom as measured in proper co-ordinates, then in accordance with equation (15) the period of emission as measured in the co-ordinates of interest will be

$$dt_e = \frac{k}{1-r^2/R^2}\, ds\,. \tag{42}$$

During this period, the source will move the radial distance

$$dr = \frac{dr}{dt}\, dt_e = \frac{k}{1-r^2/R^2}\,\frac{dr}{dt}\, ds\,, \tag{43}$$

and, in view of equation (25) for the velocity of light, the time taken for light to traverse this distance will be

$$dt_e = \frac{k}{(1-r^2/R^2)^2}\,\frac{dr}{dt}\, ds\,. \tag{44}$$

Hence the period of the light arriving at the origin will be

$$dt_a = \frac{k}{1-r^2/R^2}\, ds \pm \frac{k}{(1-r^2/R^2)^2}\,\frac{dr}{dt}\, ds\,, \tag{45}$$

where the plus sign applies to receding objects and the minus sign to approaching ones.

The period dt_a, however, is proportional to the wave-length observed at the origin and ds to the unshifted wave-length, so that we may re-write equation (45) in the form

$$\frac{\lambda+\delta\lambda}{\lambda} = \frac{k}{1-r^2/R^2} \pm \frac{k}{(1-r^2/R^2)^2}\,\frac{dr}{dt}\,, \tag{46}$$

$$\frac{\delta\lambda}{\lambda} = \frac{k}{1-r^2/R^2} \pm \frac{k}{(1-r^2/R^2)^2}\,\frac{dr}{dt} - 1\,. \tag{47}$$

If we desire we may substitute for the velocity of the source the expression given by equation (22) and write

$$\frac{\delta\lambda}{\lambda} = \frac{k \pm \sqrt{k^2 - 1 + \frac{r^2}{R^2} - \frac{h^2}{r^2} + \frac{h^2}{R^2}}}{1-r^2/R^2} - 1\,, \tag{48}$$

which gives the Doppler shift in terms of the parameters for the orbit and the distance alone.

In all the foregoing formulae r is the distance from the source to the observer and dr/dt is the velocity of the source, both taken at the time of emission and measured in a system of co-ordinates which has the observer at the origin.

11. *The Doppler shift (source at the origin).*—A somewhat different formula is obtained if we use a system of co-ordinates with the source at the origin. Let dt_e be the period of the source, dt_a the period of the arriving light, and r the distance from the source to observer at the time of arrival of the light, all measured in the foregoing co-ordinate system. Then if dr/dt is the speed of the observer at the time of arrival of the light, and, in accordance with equation (25), the speed of light is $(1-r^2/R^2)$, we may evidently connect the foregoing quantities by the equation

$$dt_a = dt_e \pm \frac{1}{1-r^2/R^2}\frac{dr}{dt}\,dt_a\,, \tag{49}$$

where the upper sign refers to a receding observer.

The quantity dt_a, however, is the period of the arriving light in terms of our present co-ordinate system and not in terms of the proper time of the observer. Hence, with the help of equation (15), we shall re-write equation (49) in the form

$$\left(1 \mp \frac{1}{1-r^2/R^2}\frac{dr}{dt}\right)\frac{k}{1-r^2/R^2}\,ds = dt_e\,, \tag{50}$$

where ds is the period of the oncoming light as measured by the observer. Noting that ds is proportional to the wave-length of the light as measured by the observer and dt_e to the unshifted wave-length, we may now re-write equation (50) in the forms

$$\frac{\lambda+\delta\lambda}{\lambda} = \frac{1-r^2/R^2}{k\left(1 \mp \dfrac{1}{1-r^2/R^2}\dfrac{dr}{dt}\right)}\,, \tag{51}$$

$$\frac{\delta\lambda}{\lambda} = \frac{1-r^2/R^2}{k\left(1 \mp \dfrac{1}{1-r^2/R^2}\dfrac{dr}{dt}\right)} - 1\,, \tag{52}$$

$$= \frac{1-r^2/R^2}{k \mp \sqrt{k^2-1+\dfrac{r^2}{R^2}-\dfrac{h^2}{r^2}+\dfrac{h^2}{R^2}}} - 1\,. \tag{53}$$

In the formulae of this section, r is the distance from source to observer and dr/dt the velocity of the observer, both taken at the time of arrival of the light and in a system of co-ordinates with the source at the origin. In this same system of co-ordinates k and h are parameters determining the motion of the observer.

For the correlation of astronomical observations the formulae of the preceding section seem to be superior to those immediately foregoing, since they make it possible to consider a single co-ordinate system with the observer at the origin and sources located and moving as desired, instead of furnishing a different co-ordinate system for each source.

Setting the value of h in equation (53) equal to zero, which is equivalent to restricting ourselves to purely radial motions, and introducing the variable χ in accordance with the transformation equation (5), we can re-write equation (53) in the form

$$\frac{\delta\lambda}{\lambda}=k\left[1\pm\sqrt{1-\frac{\cos^2\chi}{k^2}}\right]-1\,, \tag{54}$$

which is the formula obtained and used by Silberstein (*loc. cit.*) for the interpretation of observations.[1]

12. *The Doppler effect at perihelion.*—Returning now to equation (47) in section 10, and putting dr/dt equal to zero, we can obtain an expression for the Doppler effect, if the source is at perihelion at the time of emission. The result is

$$\frac{\delta\lambda}{\lambda}=\frac{k}{1-r_m^2/R^2}-1\,, \tag{55}$$

where r_m is the distance from the origin to perihelion.

By substituting the value for the parameter k in terms of h and

[1] As far as the writer is aware, equations (46), (47), (48), (51), (52), and (53) give for the first time completely accurate expressions for the Doppler shift in the De Sitter universe. Silberstein's formula (54) is correct for the very special case $h=0$, but is true in very unfortunate systems of co-ordinates which would be different for each object observed. It should further be remarked that Silberstein's applications of (54) were mostly made with the further assumption that $k=1$.

The present writer has not been able to follow the derivation of Weyl's formula (*loc. cit.*) which in our symbols appears to be $\delta\lambda/\lambda=\tan\chi$, and it certainly is not applicable to the general case of observer and source in any kind of relative motion.

r_m as given by equation (18), this expression for the Doppler effect at perihelion can be re-written in the form

$$\frac{\delta\lambda}{\lambda}=\frac{\sqrt{1-\frac{r_m^2}{R^2}+\frac{h^2}{r_m^2}-\frac{h^2}{R^2}}}{1-r_m^2/R^2}-1\,. \tag{56}$$

Hence since h^2 cannot be negative and R^2 is larger than r_m^2, we can write the inequality

$$\frac{\delta\lambda}{\lambda}\geqq\frac{1}{\sqrt{1-r_m^2/R^2}}-1\,, \tag{57}$$

or expanding the expression on the right-hand side with the neglect of terms of higher order,

$$\frac{\delta\lambda}{\lambda}\geqq\frac{1}{2}\frac{r_m^2}{R^2}\,. \tag{58}$$

We thus see that the Doppler effect at perihelion is positive and would ordinarily be interpreted as due to a motion of recession, even though the source has no radial velocity at the time of emission. The production of a positive Doppler effect by a source which is at rest was the original reason which led investigators to hope that the De Sitter universe might furnish an explanation for the preponderating shift toward the red in the light from the extra-galactic nebulae.

13. *The conditions for reversed Doppler effect.*—If now we consider a source of light which is approaching perihelion, it is evident from equation (47) that the Doppler effect will be negative as long as the second term on the right-hand side (the negative sign corresponding to approach) is large enough to outweigh the preceding term. As the source approaches nearer to perihelion, however, a point will be reached before perihelion where the Doppler effect becomes zero, and from there on to perihelion the Doppler effect will be positive although the velocity will still be negative.

To investigate the range in which this reversed Doppler effect persists, we may set $\delta\lambda/\lambda$ equal to zero in equation (48) and obtain after some rearrangement of terms

$$\frac{r_r^2}{R^2}-\frac{h^2}{r_r^2}=2-2k\,, \tag{59}$$

as the condition for the radius r_r at which reversal in the Doppler effect takes place. On the other hand, from equation (18) we have as the condition of perihelion

$$\frac{r_m^2}{R^2}-\frac{h^2}{r_m^2}=1-k^2-\frac{h^2}{R^2}, \tag{60}$$

and a reversed Doppler effect will be produced when an approaching source lies in the range between r_r and r_m.

Since our later investigations will show that k would be of the order unity and h^2/R^2 of the order zero for the objects of interest, the foregoing equations indicate a small range of distance r_r to r_m in which the reversed Doppler effect persists.

The following table, calculated from equations (59) and (60), gives values of the parameters k and h and the distance r_r, at which

TABLE I

CALCULATED FROM EQUATIONS (59) AND (60)

k	h^2	r_r^2/r_m^2
$\sqrt{1-r^2/R^2}$	0	$1+\frac{1}{4}\,r_m^2/R^2$
1	$\frac{r_m^4}{R^2-r_m^2}$	$1+\frac{1}{2}\,r_m^2/R^2$
$1+\frac{1}{2}\,r_m^2/R^2$	$\frac{2r_m^4}{R^2-r_m^2}\left(1+\frac{1}{8}\frac{r_m^2}{R^2}\right)$	$1+\frac{3}{4}\,r_m^2/R^2$
$1+r^2/R^2$	$\frac{3r_m^4}{R^2-r_m^2}\left(1+\frac{1}{3}\frac{r_m^2}{R^2}\right)$	$1+r_m^2/R^2$
$1+2r_m^2/R^2$	$\frac{5r_m^4}{R^2-r_m^2}\left(1+\frac{4}{5}\frac{r_m^2}{R^2}\right)$	$1+\frac{3}{2}\,r_m^2/R^2$
$1+4r^2/R^2$	$\frac{9r_m^4}{R^2-r_m^2}\left(1+\frac{16}{9}\frac{r_m^2}{R^2}\right)$	$1+\frac{5}{2}\,r_m^2/R^2$

reversal in the Doppler effect takes place, expressed in terms of the distance to perihelion r_m. The values of r_r^2/r_m^2 are correct only to terms of the order r_m^2/R^2. The results show, as stated, that the range r_r–r_m in which we get the reversed Doppler effect will be small provided k is of the order unity.

Although these results indicate a small range of distance in which an oncoming source gives a reversed Doppler effect, they do not necessarily indicate a small time interval for the production of

the reversed Doppler effect, since the source moves most slowly in the neighborhood of perihelion. For the case of purely radial motion, however, we can calculate with the help of equation (30) the relative length of time during which an oncoming source would produce a reversed Doppler effect, provided we assign values for the distance to perihelion and for the distance at which the source first comes under observation.

The results of such a calculation are collected below in Table II. The values in the first row are taken from Table I and correspond to pure radial motion. The radius r_0 at which the oncoming source comes under observation is taken as twice the distance to perihelion. And the distance to perihelion is arbitrarily taken as one-tenth of the distance to the horizon, which would mean a Doppler shift at perihelion, $\delta\lambda/\lambda = 0.005$.

TABLE II

CALCULATED FROM EQUATION (30)

$k = \sqrt{1 - r_m^2/R^2}$	$h = 0$	$\frac{r_r^2}{r_m^2} = 1 + \frac{1}{4}\frac{r_m^2}{R^2}$
$r_0 = 2r_m$	$r_m = 0.1R$	$\frac{\Delta t_r}{\Delta t_0} = 0.038$

The last figure in Table II is the fraction of the time of approach to perihelion during which the source would lie in the range which produces a reversed Doppler effect.

It should be noted that the fraction of the time during which the reversed Doppler effect would be observed would be slightly greater than the foregoing figure, owing to the greater length of time that it takes light to reach the origin from r_0 than from r_r and r_m. The correction, however, is not important.

It should also be noted that the result has been calculated solely for the case of purely radial motion. Similar calculations for cases of combined radial and angular motions could be made but would be tedious, owing to the complicated form of the integrated expression which would have to be used for the time of radial travel. An idea of the results to be expected can be obtained, however, from a comparison of the values of r_m, r_r, and r_0 and the velocities of radial

travel at these radii for the two extreme cases 1 and 6 given in Table I. This comparison is given below in Table III. The calculations have been made with the help of equation (23), on the assumption that terms of order r^2/R^2 can be neglected in comparison with unity.

In accordance with Table III, an oncoming source under the assumptions of case 6 would enter the range of observation with about twice as great a radial velocity as for case 1, and would have practically the same radial distance to travel to reach the radius where reversal of the Doppler effect would occur. Here the radial velocity for both cases would have fallen to a very low value, but

TABLE III

CALCULATED FROM EQUATION (23)

Case	r_o	r_r	$\frac{dr}{dt}$ at r_o	$\frac{dr}{dt}$ at r_r	$\frac{dr}{dt}$ at r_m
1.....	$2r_m$	$r_m\left(1+\frac{1}{8}\frac{r_m^2}{R^2}\right)$	$1.73\,\frac{r_m}{R}$	$\frac{1}{2}\frac{r_m^2}{R^2}$	0
6.....	$2r_m$	$r_m\left(1+\frac{10}{8}\frac{r_m^2}{R^2}\right)$	$3.12\,\frac{r_m}{R}$	$\frac{10}{2}\frac{r_m^2}{R^2}$	0

would be ten times as great for case 6 as for case 1, and this is to be compared with the conclusion that the radial distance of travel from reversal to perihelion is ten times as great for case 6 as for case 1. We may conclude somewhat roughly that the fraction of the time for case 6 during which the source would lie in the range which produces a reversed Doppler effect might be about twice as great as that given in Table II for case 1.

The foregoing leads to the general impression that a source moving with values of the parameters of the order assumed in Table I, and in such a way as to make perihelion well within the range of observation, would lie in the range to give a reversed Doppler effect for only a short part of the total time of approach, provided we take the perihelion as not more than one-tenth the radius to the horizon.

PART III. RELATION TO ASTRONOMICAL FINDINGS

14. *The facts to be considered.*—We shall now make a preliminary attempt to discuss actual astronomical findings concerning the be-

havior of the extra-galactic nebulae, under the assumption that these nebulae can be considered as free particles moving in the space-time manifold given by the De Sitter line element. Further astronomical observations and, as will appear in what follows, a further extension of the theoretical development, would be desirable for any final treatment of the correspondence with the De Sitter picture of the universe. Nevertheless, a preliminary discussion will be useful in indicating future lines for observational and theoretical work.

To carry out the proposed discussion, we must first assume certain facts concerning the nebulae as sufficiently well established so that we can provisionally regard them as correct, and see how well they fit into the De Sitter scheme. For this purpose the pertinent facts appear to be: (*a*) There is an approximately uniform distribution of nebulae in the region of space accessible to observation. (*b*) There is a great preponderance of positive Doppler effects for the nebulae, with negative effects for only a few of the nearer objects. (*c*) There is a correlation between Doppler effect and distance such that the average Doppler effect, taken at a given distance, appears to be approximately proportional to the first power of that distance in accordance with the equation

$$\left(\frac{\delta\lambda}{\lambda}\right)_{\text{av.}} = \alpha \frac{r}{R} , \tag{61}$$

the constant value α/R being of the order of 5.1×10^{-10} reciprocal light-years, and the maximum Doppler effect so far observed of the order $\delta\lambda/\lambda = 0.012$.

15. *Uniform distribution and the tendency to scatter.*—Let us first turn our attention to the distribution of the nebulae in space. On the one hand, we have the observational fact of an approximately uniform distribution of nebulae within our present range of observation. On the other hand, if we accept the De Sitter universe, we have a theoretical tendency for the nebulae to scatter, since we saw in section 6 that particles which have once reached perihelion and started to move away from an observer located at the origin will never again return to that neighborhood.

The tendency to scatter combined with the fact that we can see nebulae at all necessarily leads either to the conclusion that the

nebulae now under examination have entered the range of observation from the outside in recent enough time so that they have not yet been dispersed, or to the conclusion that these nebulae have actually been formed or created relatively recently within that range. The additional fact of approximately uniform distribution makes it appear probable that the process which has provided the nebulae now under observation is a continuous one which is still acting to maintain an approximately uniform concentration of nebulae in the neighborhood of the origin. We are thus rather naturally led to two alternative pictures to account for the observed distribution of the nebulae, which may be descriptively labeled as the "hypothesis of continuous entry" and the "hypothesis of continuous formation."

In addition to these pictures, however, if we are willing to minimize the stress laid upon the present uniform concentration of nebulae and not demand the maintenance of this concentration, we can assume that the nebulae now observed have recently entered the range of observation as the result of a fluctuation in the general density of nebular distribution, or that they have recently been produced within that range by a process which is not necessarily continuing. We may label these the "hypothesis of a concentrating fluctuation" and the "hypothesis of recent formation."

These four hypotheses and their intercombinations would seem to exhaust the obvious possibilities that lie within the general framework of the De Sitter theory. We must now examine their inherent probability and their measure of correspondence with the further facts that are available to us.

16. *The hypothesis of continuous entry.*—

a) *Inherent probability.*—In accordance with the hypothesis of continuous entry, the uniform concentration of nebulae within our range of observation would be produced and maintained by the continuous arrival of new nebulae, approaching perihelion, to make good the deficiency caused by those which have passed perihelion and are leaving the range of observation never again to return. There seems to be no inherent unreasonableness in such a hypothesis. The equations of motion for particles in the De Sitter universe are completely

reversible,[1] and there seems to be no reason why the a priori probability for approaching and receding nebulae should not be the same. Even if we consider the neighborhood of the horizon, it seems proper to assume that the number of nebulae moving—there with great slowness—toward the horizon would be equal to the number moving away from the horizon with the same speed. In fact, the hypothesis of continuous entry appears in every way a very natural one to consider.

b) *Compatibility with the preponderance of positive Doppler effects.* —If we should assume the hypothesis of continuous entry, there would be at any time equal numbers of approaching and receding nebulae within the range of observation, and we should have to reconcile this fact with the great preponderance of positive Doppler effects, which are actually observed. It was shown in Table I in section 13, however, that an approaching nebula would give a positive Doppler effect only when it is very close to perihelion, provided r^2_m/R^2 is small and k is not greatly different from unity. It was shown by Tables II and III that this small range of distance in which the reversed Doppler effect would occur would correspond to a small fraction of the total time of approach, provided that perihelion was not too close to the limit of observation. Hence it is evident that we shall have to give special consideration to the reconciliation of the hypothesis of continuous entry with the observed preponderance of shifts toward the red.

At first sight it might seem that we could bring about the reconciliation by assuming larger values of k for the typical nebulae than those considered in Table I, or a smaller value of R than that taken in Table II. But this is not possible, since the considerations of sections 16*c* and *d* will show that we cannot greatly increase k or decrease R without getting larger Doppler effects than those actually observed.

A reconciliation can apparently be obtained, however, by intro-

[1] The recognition of the fact that the equations of motion in the De Sitter universe are reversible was one of the important contributions of Silberstein. Other investigators have often had a tendency to assume that free particles could only be moving away from the observer under the action of the repulsive force which we can think of as emanating from the origin. This, however, would be quite erroneous, as first made clear by Silberstein.

ducing a special assumption regarding the numbers of nebulae which make perihelion at different distances from the origin.

It is to be noted that the astronomer is not to be regarded as following a given nebula throughout its path and determining the fraction of the time that it gives a positive Doppler effect; but rather is to be regarded as examining the nebulae within a given range of distances r to $r+dr$ and determining the fraction of that number which give a positive Doppler effect. Now within any such range r to $r+dr$ we shall have of course, on the hypothesis of continuous entry, approximately equal numbers of approaching and receding nebulae, and hence, neglecting the time taken for light to reach us from the nebulae which produces a small effect in favor of those which are receding, we shall have at any time an approximately equal chance of seeing an approaching or a receding nebula. There is nevertheless nothing immediately evident to prevent our making the assumption that the fraction of the nebulae in this range which makes perihelion close to r is much greater than the fraction which passes through r to make perihelion at a radius r_m appreciably less than r, and hence that the majority of the approaching nebulae between r and $r+dr$ will be near enough perihelion so as to give the reversed positive Doppler effect.

Thus by assuming that the number of nebulae which make perihelion at a given radius increases very rapidly with the radius, we should be able to account for the preponderating positive Doppler effect. It must be left as a matter for further investigation to determine whether the assignment of the values of k and h to the different members of a system of nebulae in such a way as to secure the foregoing result would be compatible with the permanent maintenance of the approximately uniform concentration of nebulae now observed. In any case it can be remarked that the experimental fact of a great preponderance of positive Doppler effects might impose, on the basis of the continuous-entry hypothesis, severe requirements on the values of the parameters k and h which are to be assigned to the nebulae under observation.

c) Value of the average Doppler effect at any radius "r."—We may now turn to the correlation between average Doppler effect and distance which is to be expected on the basis of the hypothesis of

continuous entry. Since the number of approaching and receding nebulae which we observe in any shell r to $r+dr$ will be approximately equal on the foregoing hypothesis, the second term in the expression for the Doppler effect given by equation (47) will cancel out when we take the mean, and we shall obtain for the average Doppler effect at any radius r

$$\left(\frac{\delta\lambda}{\lambda}\right)_{\text{av.}} = \frac{k_{\text{av.}}}{1-r^2/R^2} - 1 \,. \tag{62}$$

This expression contains the average value of the parameter k for the nebulae which we observe at any radius r, and hence does not give a completely determined result in the absence of information as to the assignment of parameters to the whole system of nebulae. Nevertheless, we know that the value of k for any nebula which we observe at the radius r must at least be great enough so that perihelion would not occur at a greater distance from the origin than r, and hence in accordance with equation (18) we may write

$$k \geqq \sqrt{1-r^2/R^2} \,, \tag{63}$$

and substituting above we obtain

$$\left(\frac{\delta\lambda}{\lambda}\right)_{\text{av.}} \geqq \frac{1}{\sqrt{1-r^2/R^2}} - 1 \,, \tag{64}$$

or, approximately, neglecting higher powers in r/R,

$$\left(\frac{\delta\lambda}{\lambda}\right)_{\text{av.}} \geqq \frac{1}{2}\frac{r^2}{R^2} \,. \tag{65}$$

Although this inequality does not rule out a linear increase in average Doppler effect with distance, as appears to be found experimentally, it does not make such a result seem in any way inevitable.

To investigate the necessary dependence of $k_{\text{av.}}$ on r which would lead to such a linear increase, we may combine the experimental relation given by equation (61) and the theoretical relation given by equation (62) and write

$$\left(\frac{\delta\lambda}{\lambda}\right)_{\text{av.}} = \alpha\frac{r}{R} = \frac{k_{\text{av.}}}{1-r^2/R^2} - 1 \,, \tag{66}$$

and solving for $k_{\text{av.}}$ obtain

$$k_{\text{av.}} = 1 + \alpha\frac{r}{R} - \left(1+\alpha\frac{r}{R}\right)\frac{r^2}{R^2} \,. \tag{67}$$

It should be noted that this equation would impose severe further requirements on the assignment of parameters to the nebulae, in addition to those which are necessary if the hypothesis of continuous entry is to be compatible with a predominant positive Doppler effect and a permanent maintenance of an approximately uniform concentration of nebulae, within our present range of observation.

d) Values of "R" and "k."—To complete the discussion, attention should be called to the values of R and k which would be in agreement with the hypothesis of continuous entry.

Returning to equation (65) and remembering that the maximum Doppler effect so far observed is of the order 0.01, it is evident that the radius of the observable universe R must be assumed to be at least of the order of ten times the range of distances at which we now observe nebulae. Otherwise we should expect higher Doppler effects than are observed. This would make R not less than of the order of 2×10^8 light-years.

Similarly, from equation (62) it is evident that values of k greater than about 1.01 are not to be expected in our present range of observation. This explains the choice of values for R and k which were used in the calculations of section 13.

17. *The hypothesis of continuous formation.*—

a) Inherent probability.—Let us now turn to the hypothesis of continuous formation. In accordance with this hypothesis, the nebulae which we can now observe have been formed recently enough so that they have not yet escaped from our range of observation, and this process of formation is a continuing one which will permanently maintain an approximately uniform concentration of nebulae.

This idea appears to have little inherent probability. Science has not yet provided us with any clearly worked-out mechanism by which such a formation of nebulae could take place, and an appeal to special acts of creation is not within the province of our discussion. Nevertheless, we should not completely disregard the possibility that some such process—perhaps associated with a condensation of radiation into matter—might be taking place.

b) Compatibility with positive Doppler effects.—By making the hypothesis of continuous formation we place ourselves in any case on such thin speculative ice that it should not at present be distaste-

ful to us to make the further assumption that most of the nebulae we now observe, actually are receding from us, and in this way account for the preponderating red shift in the Doppler effect from these objects.

We might attempt to justify this assumption by saying that for some reason the nebulae tend to form at or near perihelion and then escape, or that they condense around particles which have been dislodged from our own system and are hence already escaping. Such justifications, however, will hardly improve our uncomfortable frame of mind.

c) Value of the average Doppler effect at any radius "r."—Finally, it is evident that we can also make *ad hoc* assumptions which would give us any desired relation between Doppler effect and distance. Thus if we take the hypothesis that the nebulae we observe have condensed around particles which have just been able to escape from our own system, we must assign to them approximately the parameter values $k=1$ and $h=0$, the same as for our own system. Then in accordance with equation (48) we should obtain for the Doppler effect

$$\frac{\delta\lambda}{\lambda}=\frac{1+r/R}{1-r^2/R^2}-1\,, \tag{68}$$

or neglecting the square of r/R,

$$\frac{\delta\lambda}{\lambda}=\frac{r}{R}\,, \tag{69}$$

which would give us the empirical, reasonably well-established linear dependence on distance. It must be emphasized, however, that equation (69) is obtained by a neglect of the negative sign in equation (48) and a rather arbitrary assignment of values to the parameters k and h.

d) Value of "R."—Combining our present theoretical equation (69) with the experimental relation given by (61), we should conclude on the basis of the hypothesis of continuous formation that the radius of the observable universe R was of the order of 2×10^9 light-years, or ten times as great as the lower limit obtained on the basis of the hypothesis of continuous entry.

18. *The hypothesis of recent formation and of a concentrating fluctuation.*—Two other possible hypotheses, which would account

for the presence of nebulae at all within the range of observation, were stated in section 15. A few words will suffice for them, since most of the features they present have already been discussed in the two previous sections.

The hypothesis of recent formation differs from that of continuous formation only in assuming that the process by which the nebulae form within the range of observation is not a continuing one. It is hence perhaps even more improbable than the latter, since it comes even nearer to the appearance of appealing to a special act of creation. By *ad hoc* assumptions it can of course also be made to give results which agree with any desired relation between Doppler effect and distance.

In accordance with the hypothesis of a concentrating fluctuation, the presence of a large concentration of nebulae within the present range of observation is the result of a fluctuation from a smaller general concentration of nebulae, which has occurred sometime in the past. The hypothesis is not entirely without merits. If we assume that we are now witnessing the dissipating phase of this fluctuation, we have a ready explanation of the predominant positive Doppler effect. If the past increase in concentration brought the nebulae well within each other's range of action, we might assume that they are now escaping with approximately the parameter values $k=1$ and $h=0$, as would be the case for a particle which is just barely dislodged from the origin. With these parameter values, we should of course again obtain the linear relation between Doppler effect and distance given by equation (69). The chief demerit of the hypothesis is its appeal to a special kind of past fluctuation in the concentration to account for the observed facts.

19. *Conclusion.*—In conclusion it certainly must be stated that the De Sitter line element for the universe does not appear to afford a simple and unmistakably evident explanation of our present knowledge of the distribution, distances, and Doppler effects for the extra-galactic nebulae.

In order to reconcile the tendency for particles to scatter in the De Sitter universe with the presence of nebulae at all within the range of observation, we found it necessary to make some hypothesis concerning the motion or origin of the nebulae. Of these hypotheses the most natural was that of continuous entry, in accordance with

which nebulae in approximately equal numbers would be continuously entering and leaving the range of observation. Although this hypothesis would lead as shown by expression (65) to an increase in average Doppler effect with distance, it would not appear to account for the very rare occurrence of negative Doppler effects and for a linear increase in average Doppler effect with distance, without the imposition of restrictions on the parameters determining the orbits of the nebulae. It remains to determine the exact nature of these restrictions and to show that they are in any way inevitable or natural.

The other hypotheses that were examined all contained varying degrees of inherent improbability, although by the introduction of *ad hoc* assumptions they could be made to give any desired relation between Doppler effect and distance.

Further observational material on the nebulae would be of great importance. In particular it is desirable to be certain as to the form of the relation between Doppler effect and distance, and also as to the relative frequency of negative and positive Doppler effects.

Further theoretical work on the De Sitter universe would also be of interest. An investigation of the probable distribution of values of the parameters k and h among the nebulae would be important. In addition it should be specially noted that the applications made in this article have all been based on the assumption that the motions of the extra-galactic nebulae can be treated as approximately those of free particles in a universe having exactly the De Sitter line element. The De Sitter line element, however, can be regarded as derived from the postulate of a completely empty universe, and the extent of distortion away from the exact form given by De Sitter produced by the actual presence of matter could well stand further investigation.[1]

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[1] Some treatment of this problem has already been given by Silberstein (*loc. cit.*), who concluded in the first place that the Doppler effect would not be appreciably modified by the gravitation of our galactic system, and in the second place that the effect of the De Sitter repulsion would become greater than the ordinary gravitational attraction of our own galaxy even within the limits of that galaxy.

ON THE PHYSICAL INTERPRETATION OF SPECTROHELIOGRAMS[1]

By ALBRECHT UNSÖLD[2]

ABSTRACT

Introduction.—Short recapitulation of the observations.

1. *Radiative equilibrium in a stratified atmosphere with scattering and absorption.*—The intensity of radiation emerging from such an atmosphere is calculated as a function of the coefficients of scattering and of absorption and of the temperatures of the different layers.

2. *Application of these theoretical considerations to calcium spectroheliograms.*—It is shown how different parts of the lines are co-ordinated with different levels. Photometric measures of the distribution of intensity in the inner parts of the H and K lines are described.

3. *Hydrogen spectroheliograms.*—A comparison of calcium and hydrogen spectroheliograms shows that in disturbed regions of the sun the hydrogen atoms are probably affected through electric disturbances by other particles.

4. *In the Evershed effect* and *similar phenomena* lines of the *same multiplet* seem to indicate *different velocities*. This observation can be explained on the basis of the theory developed in sections 1 and 2, supplemented by considerations on contrast effect. Experiments with artificial lines confirm this explanation.

INTRODUCTION

The spectroheliographic researches of G. E. Hale[3] and of H. Deslandres[4] led to the important result that spectroheliograms taken with the center of a Ca^+ line correspond to the highest calcium levels of the solar atmosphere, while pictures in the light of the wings of these lines represent layers whose depth increases with increasing distance from the center of the line. This interpretation was based on the general appearance of the pictures and has been confirmed by the observations of Adams[5] on the differences in the rotational velocities of the outer solar layers, and of St. John[6] on the Evershed effect. The physical interpretation of these relations, however, re-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 378.

[2] Fellow of the International Education Board.

[3] *Yerkes Observatory Publications*, 3, 1903, and *The Study of Stellar Evolution*, Chicago, 1908.

[4] *Annales de l'Observatoire de Paris-Meudon*, 4, 1910.

[5] *Mt. Wilson Contr.*, No. 33; *Astrophysical Journal*, 29, 110, 1909.

[6] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, 37, 322, 1913.

mained in an unsatisfactory state. As long as pressures of several atmospheres were ascribed to the reversing layer, direct effects of pressure could be assumed, and in this way the wings of the lines could be attributed to the most "quenched" atoms of the deeper layers. Since the pressures in solar and stellar atmospheres have been shown by both observation[1] and general astrophysical considerations to be very low, this explanation is generally discarded today. After it was realized[2] that the H and K lines are produced mainly by resonance scattering, the following explanation might have been substituted: As the coefficient of scattering is considerably greater for the light of the middle of the line than for that of the wings, it might have been assumed that the spectroheliograph penetrates deeper into the solar atmosphere with the light of the wings than with the light of the center of the line, just as a photographic camera penetrates farther into a hazy atmosphere with a red filter than without. This comparison, however, cannot be applied directly to the sun, for the exchange of radiation in the solar atmosphere is regulated in a manner wholly different from that in the terrestrial atmosphere. Furthermore, on the earth the objects to be seen are small compared with the dimensions of the atmosphere, while on the sun this is usually not the case. Moreover, it can easily be shown from known formulae that an increase of the density in a certain layer influences (in the case of pure resonance scattering) the different parts of a line in exactly the same manner. As the changes in the form of spectral lines important for us concern especially the middle of these lines, it is reasonable to assume that in the present problem atomic collisions play a decisive rôle. To clarify the matter, we shall therefore (sec. 1) first apply the theory of radiative equilibrium to a stratified atmosphere in which occur resonance scattering and collisions, the collisions producing absorption in the sense in which the term was used by Schwarzschild. These considerations lead to a satisfactory theory of spectroheliograms (sec. 2). In sections 3 and 4 we shall briefly treat the differences between calcium and hydrogen spectroheliograms, and, finally, discuss a curious observation con-

[1] C. E. St. John and H. D. Babcock, *Mt. Wilson Contr.*, No. 278; *Astrophysical Journal*, **60**, 32, 1924.

[2] Cf., for instance, A. Unsöld, *Zeitschrift für Physik*, **46**, 765, 1928.

cerning currents in different solar layers which is connected with the considerations of section 1.

1. RADIATIVE EQUILIBRIUM OF A STRATIFIED ATMOSPHERE WITH SCATTERING AND ABSORPTION[1]

We consider the following schematic solar atmosphere:

Above the photosphere let there be, first, the layer I (reversing layer), with the scattering and absorption coefficients σ, and κ_1 and the Kirchhoff-Planck function B_1, all of which are assumed to be constant throughout this layer. For the sake of simplicity we consider the emission of the photosphere, I_0, as independent of direction. Above this assume a second layer II, in which these quantities have the values σ_2, κ_2, and B_2. We shall later on identify this layer with the upper part of the reversing layer, or the chromosphere.

In order to simplify the calculations, we assume furthermore the inward radiations, I_1' and I_2', for layers I and II, respectively, and the outward, I_1 and I_2, to be independent of direction. The differential equations of radiative equilibrium have then the well-known Schwarzschild-Schuster form. Their solutions for layers I and II may therefore be taken from my earlier paper.[2]

We have for layer I:

$$\left.\begin{aligned} 2I_1 &= R_1(1-\omega_1)e^{-u_1} + S_1\left(\frac{e^{u_1}-e^{-u_1}}{a_1} + \omega_1\frac{e^{u_1}+e^{-u_1}}{a_1}\right) \\ &\qquad + 2B_1\left(1-\omega_1\frac{e^{u_1}-e^{-u_1}}{2} - \frac{e^{u_1}+e^{-u_1}}{2}\right), \\ 2I_1' &= R_1(1+\omega_1)e^{-u_1} + S_1\left(\frac{e^{u_1}-e^{-u_1}}{a_1} - \omega_1\frac{e^{u_1}+e^{-u_1}}{a_1}\right) \\ &\qquad + 2B_1\left(1+\omega_1\frac{e^{u_1}-e^{-u_1}}{2} - \frac{e^{u_1}+e^{-u_1}}{2}\right), \end{aligned}\right\} \quad (1)$$

[1] The following calculations are very closely connected with the former paper: "Über den Einfluss von Stössen auf die Struktur der Fraunhoferschen Linien," *Probleme der modernen Physik: Festschrift zum 60. Geburtstag von A. Sommerfeld*, Leipzig, 1928. The notations are essentially the same. Here we shall repeat only the most important principles.

[2] *Op. cit.*, equation (7).

and correspondingly for layer II:

$$\left.\begin{aligned} 2I_2 &= R_2(1-\omega_2)e^{-u_2}+S_2\left(\frac{e^{u_2}-e^{-u_2}}{a_2}+\omega_2\frac{e^{u_2}+e^{-u_2}}{a_2}\right) \\ &\qquad +2B_2\left(1-\omega_2\frac{e^{u_2}-e^{-u_2}}{2}-\frac{e^{u_2}+e^{-u_2}}{2}\right), \\ 2I_2' &= R_2(1+\omega_2)e^{-u_2}+S_2\left(\frac{e^{u_2}-e^{-u_2}}{a_2}-\omega_2\frac{e^{u_2}+e^{-u_2}}{a_2}\right) \\ &\qquad +2B_2\left(1+\omega_2\frac{e^{u_2}-e^{-u_2}}{2}-\frac{e^{u_2}+e^{-u_2}}{2}\right). \end{aligned}\right\} \quad (2)$$

The notation is the same as in the former paper. In formulae which are equally valid for layers I and II, the subscripts 1 and 2 will be omitted. R and S are constants of integration, which will subsequently be determined. If we set $\frac{2K}{\sigma}=\lambda$ and $t=\frac{3}{4}\sigma\times$depth, we have, as in the previous paper, $u=\sqrt{\lambda(2+\lambda)}\,t$, $a=\sqrt{\lambda(2+\lambda)}$, and $\omega=\sqrt{\frac{\lambda}{2+\lambda}}$.

The boundary conditions which are necessary for the determination of the four constants of integration can now be formulated as follows:

A. At the outer boundary of the atmosphere the inward radiation disappears.

B. At its inner boundary, layer I receives the radiation of the photosphere; that is, for $u_1=u_{10}$, a given large value (compare, however, condition C), we have $I_1=I_0$.

C. At the boundary between layers I and II, $I_1=I_2$ and $I_1'=I_2'$. The depth $u_1=u_{12}$ corresponding to the upper boundary of layer I remains undetermined, since in the final equations u_{12} occurs in such relation to the constants of integration that only the difference $u_{12}-u_{10}$ has a physical meaning.

Condition A, by means of equation (2), gives at once:

$$R_2(1+\omega_2)-S_2\frac{2\omega_2}{a_2}=0. \quad (3)$$

To obtain the second boundary condition in a convenient form and to simplify the following calculations, we assume that the op-

tical depth of layer I is always >1, which means that we confine ourselves to following the changes of the broad H and K lines in the neighborhood of their centers. Then we obtain, with the aid of (1):

$$S_1 = a_1 B_1 . \tag{4}$$

The boundary conditions (C), by means of (1) to (4), may now be written, the lower boundary of layer II being determined by u_2:

$$\left.\begin{aligned}(1-\omega_1)(R_1-2B_1)e^{-u_{11}}+2B_1 &= R_2\left\{2\cosh u_2+\left(\omega_2+\frac{1}{\omega_2}\right)\sinh u_2\right\}\\ &\quad +2B_2\{1-\omega_2\sinh u_2-\cosh u_2\},\\ (1+\omega_1)(R_1-2B_1)e^{-u_{11}}+2B_1 &= -R_2\left(\omega_2-\frac{1}{\omega_2}\right)\sinh u_2\\ &\quad +2B_2\{1+\omega_2\sinh u_2-\cosh u_2\}.\end{aligned}\right\} \tag{5}$$

By elimination of R_1, the constant R_2 can be calculated. We find:

$$\left.\begin{aligned}R_2\left\{(1+\omega_1)\cosh u_2+\left(\omega_2+\frac{\omega_1}{\omega_2}\right)\sinh u_2\right\}\\ =2(B_1-B_2)\omega_1+2B_2(\omega_1\cosh u_2+\omega_2\sinh u_2).\end{aligned}\right\} \tag{6}$$

Now we are already at our goal, since by means of (2) we see that at the outer boundary of the atmosphere

$$I_2 = R_2 . \tag{7}$$

In other words, the constant of integration, R_2, itself gives directly the average intensity of the observed radiation. Its distribution in direction we shall not treat.

2. INTERPRETATION OF CALCIUM SPECTROHELIOGRAMS

In order to obtain a physical interpretation of the calcium spectroheliograms, we must discuss in what way the shape of the line depends upon the constants ω_1, B_1 and ω_2, B_2.

If we leave layer II out of consideration for the moment ($u_2 = 0$), equation (6) gives for the residual intensity of the primary absorption line the value (for integrated radiation)

$$R_2 = R_1 = \frac{2B_1\omega_1}{1+\omega_1}, \tag{8}$$

which was contained implicitly in the above-mentioned paper. If, however, $u_2 \gg 1$, that is, if the optical depth corresponding to the middle of our "secondary" line is still considerable, we have

$$R_2 = \frac{2B_2\omega_2}{1+\omega_2}, \tag{9}$$

which shows that the residual intensity then depends only on the uppermost layer. This means that, roughly speaking, the radiation of deeper layers cannot in an appreciable degree penetrate through this layer. However, if $u_2 \ll 1$, we can develop equation (6) in increasing powers of u_2 and obtain[1]

$$R_2 = \frac{2(B_1\omega_1 + B_2\omega_2 u_2)}{1+\omega_1+\left(\omega_2+\frac{\omega_1}{\omega_2}\right)u_2} = R_1\left\{1-\left(\frac{\omega_2+\frac{\omega_1}{\omega_2}}{1+\omega_1}-\frac{B_2\omega_2}{B_1\omega_1}\right)u_2\right\}. \tag{10}$$

In this case the two lines are superposed and the upper layer gives only a comparatively small modification of the center of the line.

For the Ca^+ lines our second assumption ($u_2 < 1$) should come nearer to the truth.

Let us assume now that, through deviations from equilibrium conditions in the lower layer I, the constant B_1, or ω_1, or both together, rise above their average values. Then R_1 will increase according to (8). The more we approach the center of the line, the greater will be the increase in brightness indicated by the spectroheliograph. In the very schematic Figure 1 let the line a correspond to undisturbed conditions. Then line b will result from the described changes. For, as shown in the earlier paper, the intensity of the outermost part of the wings does not depend at all upon B_1 and ω_1, while the intensity of the intermediate and inner parts is determined by these very factors. This effect is modified, however, as soon as the zone of the secondary line, originating in layer II, is entered. If the u_2 corresponding to the middle of this line is $\gg 1$,

[1] For $B_2 = 0$ and $\lambda_2 \to 0$ we get from this formula at once the one (equation 2′) used in the preceding paper (*Mt. Wilson Contr.*, No. 377; *Astrophysical Journal*, **69**, 209, 1929).

we shall, according to (9), have there no change at all. If $u_2 \ll 1$, then according to (10) the effect will not increase in any case (cf. again Fig. 1, curve *b*).

Just the reverse happens if we change the constants B_2 and ω_2 of the upper layer, but leave unchanged those of the lower one. Then R_1, and therefore the principal lines, naturally remain unmodified, while the secondary line changes according to (10). In Figure 1 these modifications are represented in various degrees by curves *c* and *c'*.

We observe that the behavior of the whole line corresponds throughout to the spectroheliographic observations. The bright

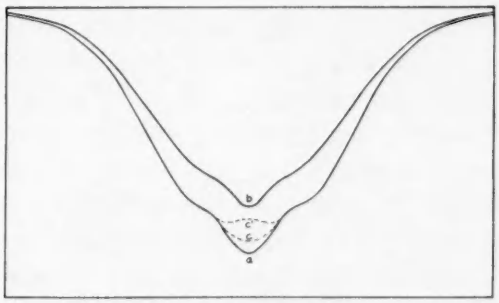

FIG. 1.—Schematic Contours

parts correspond therefore to an increase either of the (approximate) Kirchhoff function B_2 or of the activity of the collisions, ω_2.

Since the optical depth of the higher Ca^+ layers is rather small, we must have $u_2 < 1$. It follows from this, in accordance with the preceding considerations, that all the flocculi which are visible in the light of the wings appear also in the spectroheliograms taken with the center of the line.

We have still to write down the theoretical conditions corresponding to real emission (H_2, K_2) in the middle of the Ca^+ absorption lines. Equation (10) shows that if the intensity in the line is to increase with the optical depth (toward the line-center),

$$\frac{\omega_2 + \dfrac{\omega_1}{\omega_2}}{1+\omega_1} - \frac{B_2\omega_2}{B_1\omega_1} < 0 . \qquad (11)$$

That is, either $\omega_2 > \omega_1$, $B_2 > B_1$ or both. The presence of emission lines is therefore connected with a reversal of the temperature- or pressure-gradient (ω increases with pressure).

From our discussion it appears that the preceding considerations on the relations of the different parts of the calcium lines are applicable to both absorption and emission lines.

To summarize, the theory of radiative equilibrium therefore leads to the following picture of the solar calcium atmosphere: The broad H_1 and K_1 lines originate at a depth where collisions play a part not to be neglected (as was shown previously, $\lambda_1 = 0.088$ and $\omega_1 = 0.2$). Above this comes a transition layer, in which, apparently, abnormally strong excitation by collisions takes place very easily. This excitation seems to be connected[1] with the very rapid change in density between reversing layer and chromosphere; besides this, the "aerodynamic" perturbations in the neighborhood of spots and faculae surely play an important rôle. In the outer chromosphere, finally, collisions cannot be of any importance. A strong absorption (H_3, K_3) therefore corresponds to this layer. Quite generally, the innermost parts of a line correspond to the highest layers, the outer parts to deeper layers. The form of the outermost wings, in which, as previously shown, collisions are not of importance, is determined only by the number of atoms in the whole atmosphere.

In order to get an idea of the quantitative side of the phenomena, two spectrograms of the integrated solar radiation taken November 9, 1928, with the 75-foot spectrograph of the 150-foot tower telescope, without the objective, were measured photometrically. It was found that the maxima of the H_2 and K_2 lines rise only about 6 per cent above the brightness of the surrounding minima. The strong visual impression produced by these lines is much exaggerated by contrast effect. For the sun as a whole the average deviations from the state of equilibrium are therefore rather small; in the neighborhood of disturbed regions they are naturally much greater. The intensity in the middle of the H_3 and K_3 lines came out about 23 per cent smaller than the intensity of the H_2 and K_2 maxima, in close agreement with measures in the light from the center of the solar disk.

[1] Cf. R. W. Gurney, *Monthly Notices, R.A.S.*, **88**, 377, 1928.

3. HYDROGEN SPECTROHELIOGRAMS

As the well-known work of Hale and of Deslandres has shown, spectroheliograms taken with the hydrogen lines, especially *Hα*, give pictures which are essentially different from the calcium pictures.[1] Photographs with the border of the *Hα* line usually show dark hydrogen flocculi, instead of bright flocculi. Photographs taken with the middle of the line show similarity to those obtained in H_3 or K_3 light, but the pictures are simpler. This anomalous behavior of the hydrogen lines is not astonishing, as the contours of the undisturbed Balmer lines are very different from those of other lines and are therefore likely to have a different physical cause. As has already been assumed by various authors, electrical disturbances by neighboring particles—a kind of Stark effect—are likely to play the decisive rôle.

On the other hand, as Hale and Deslandres have pointed out, the difference between calcium and hydrogen flocculi arises from the fact that to an enhancement of the H_2 and K_2 emissions there always corresponds a broadening of the hydrogen lines, which in many cases is apparently connected with an increase of the central intensity. A series of *Hα* and H+K spectra taken in immediate succession in November, 1928, confirms the correlation between the width of *Hα* and the brightness of H_2 and K_2. Photometric measures show, moreover, that *Hα* never becomes narrower than it is normally, but only broader (in disturbed regions). The change in central intensity does not seem to be uniquely connected with the change of line width; but the spectroheliographic results do favor a rather strong correlation between the two phenomena.

An explanation of the phenomena just described is as follows: As we have seen, the brightening of the H_2 and K_2 lines is caused by increased excitation by collisions. If we assume now, as is very plausible, that the active particles are charged electrically—in other words, that they are ions or free electrons—we can understand at once that, as in a condensed discharge, the hydrogen lines may be broadened by their electrical fields. In the inner part of the line, however, according to our general theory, the dependence of the

[1] Cf. the extremely interesting reproductions in the paper by H. Deslandres, already cited, especially Plate 36*bis*.

coefficients of scattering and absorption on wave-length is rather unimportant; the middle of the line will therefore behave nearly in a normal way, as the observations indicate. The assumption that perturbed parts of the solar surface contain a considerable number of free charges is strongly favored by the existence of strong magnetic fields in the neighborhood of spots.

4. EVERSHED EFFECT AND LIMB-CENTER DISPLACEMENTS

After having cleared up theoretically the relations of the various zones of a spectral line to the stratification of the solar atmosphere, some remarks concerning a curious phenomenon connected with the currents of the solar atmosphere may be in order.

According to the well-known observations of Evershed and of St. John, the velocity of the currents in the solar atmosphere in the neighborhood of spots (Evershed effect) and also the general currents (limb-center displacements)[1] show a marked dependence on the height above the photosphere.

It seems, however, a perfect riddle that not only various atoms, but also lines of the same multiplet and other groups of lines, which originate in the same atomic level, indicate different velocities, so that strong (broad) lines may correspond to a higher level than weak (narrow) lines. This observation seems at first sight to contradict all atomic physics. We shall show, however, that the general results of section 2, connected with some elementary facts from physiological optics,[2] which naturally influence all such measurements, give at once an explanation.

As we have seen, the core of a line corresponds, roughly speaking, to high layers, the outer parts to deep layers.

On the other hand, the visual impression of a line, whether observed directly or on the photographic plate, depends not only on the true distribution of intensity, but is also strongly influenced by the effect of contrast. This in our case is approximately proportional to the second derivative of the intensity with respect to the longitudinal extension of the spectrum, as measured on the retina of the observing eye.[2] From this it follows that, other things

[1] Our further considerations would still remain valid, even if these displacements are to be explained by something else than Doppler effect.

[2] For the following cf. the paper by A. Kühl, *Physikalische Zeitschrift*, **29**, 1, 1928, which is extremely important for all such questions. Further references are given there.

being equal, the contrast effect for similar lines, that is, lines whose widths for every value of intensity are in a constant ratio, is inversely proportional to the square of line width. The contrast effect for a broad line will therefore be rather unimportant, and in setting the cross-wire one will be likely to put it on the real minimum of the line.

In the case of a narrow line, however, the "center" is determined primarily by the contrast effect, which is connected with the rapid change in the slope of the intensity-curve; the real intensity-distribution plays a subordinate part. In this case the cross-wire will be set not on the real minimum but about halfway between the points of the most rapid change in intensity.

If the various layers of the solar atmosphere have different velocities, the Fraunhofer lines will be somewhat asymmetrical,[1] because the minimum indicates the velocity of the highest layers, while the displacement of the outer parts of a line corresponds to deeper layers. From the previous considerations concerning physiological optics it is now apparent why measures of narrow lines give the velocity of deep layers, but broad lines the velocity of high layers, although the real displacements of the various parts of both lines are the same.

In order to test this theory experiments with artificial asymmetrical lines were made as follows: Profiles of the lines, with a certain displacement of the minimum with respect to the middle of the wings, were drawn on paper. The areas within the lines were then blackened and fixed around a cylinder which could be rapidly rotated. This device affords a means of producing lines having any given distribution of intensity. Besides the lines themselves, reference marks were also made on the drawing. In order to approach the circumstances of ordinary measurement as closely as possible, the whole arrangement was photographed. The pictures of the rotating cylinder seem to show a real spectral line. Two lines were made having the same displacement of the center relative to the wings, but with widths in the ratio of 3:1, about 0.6 and 0.2 mm, respectively, on the plate. For measurements with a microscope of the usual magnifying power (about 15), the result in both cases was

[1] Cf. also K. Burns and W. F. Meggers, *Publications of the Allegheny Observatory*, 2, 105, 1910.

that the settings were made on the real minimum. With low magnifying power, however, it was apparent that in the case of the broad line the eye still selected the real minimum, but in the case of the narrow line, the settings were nearer to the middle between the wings. While in the first case the shifts, on an arbitrary scale, were 1.6 for both broad and narrow lines, they were in the second case 1.7 for the broad, but only 1.05 for the narrow line. Under the conditions of dispersion and magnifying power which occur in the measures of the solar spectrum here considered, it is likely that the limit between the two kinds of setting, which naturally is not sharply defined, falls within the range of line widths appearing on the spectrograms. The quantitative side of the phenomenon is likely to depend largely on the character of the plate and the kind of development used. For the conditions which prevailed in the experiments described and for the usual measuring microscope, the limiting line width on the plate would be of the order of 0.05–0.10 mm. Since, however, real spectral lines on plates taken for measurement for various reasons often look considerably blacker than those of the experiments, the limit may be shifted toward broader lines. The discrepancy between solar observations and the requirements of atomic physics, which at first sight appears so strange, thus finds a satisfactory explanation.[1] Further observations concerning the problems here considered are naturally very desirable.

The writer wishes to express his thanks to Dr. Charles E. St. John for much help and advice, and to the International Education Board for the opportunity of working at the Mount Wilson Observatory.

Carnegie Institution of Washington
Mount Wilson Observatory
January 1929

[1] As J. W. Nicholson and T. R. Merton, *Proceedings of the Royal Society*, A, **98**, 261, 1920, have pointed out, the finite resolving power of the spectrograph acts in the same direction.

A THEORY OF THE MAGNETIC FIELD ASSOCIATED WITH SUN-SPOTS

By ROSS GUNN

ABSTRACT

Theory of sun-spot magnetic fields based on the drift of ions produced by ions spiraling around a non-homogeneous magnetic field. The Hale-Bjerknes theory is retained almost completely. It is shown that the *small initial symmetrical magnetic field produces currents in such a direction that regeneration is possible* and that the *currents flowing in the steady state* are ample to *account for the observed fields.* The square of the *computed fields are* found to be *proportional* to a logarithmic function of the *radius* of the spot and the *depth of the conducting layer.*

Hale[1] and Bjerknes[2] have considered the relation of solar vortices to the physical properties of sun-spots. They attribute the spot to deep-seated vortices whose termini appear on the surface of the sun and there transfer angular momentum to the sun's atmosphere in such a manner as to produce a tornado. This tornado gives rise to an expansion of the gases with consequent cooling, and it is further supposed that the rotation of the ionized gas produces a separation of charge giving rise to a magnetic field. The polarity of the spot would then depend on the direction of rotation of the deep-seated vortex, and it is therefore possible to correlate the magnetic sun-spot period with an internal circulation period in the manner proposed by Bjerknes. For the purposes of this paper the theory of Hale and Bjerknes describes adequately the mass motion of the particles in the sun, and the ion vortex will be retained not to account for the magnetic field, but to provide the initial symmetrical magnetic "seed" of the correct polarity to start off the phenomena to be described.

Recent investigations by the writer of the motion of ions spiraling about inhomogeneous magnetic fields have shown that drift motions are imposed which are oppositely directed for the positive and negative ions.[3] Under conditions of radial symmetry and a closed circuit, such as exists just outside a sun-spot, a current flows

[1] *Astrophysical Journal*, **28**, 100, 1908. [2] *Ibid.*, **64**, 93, 1926.

[3] R. Gunn, "An Electromagnetic Effect of Importance in Solar and Terrestrial Magnetism," *Physical Review*, **33**, 832, 1929.

which is in such a direction as to reduce the local inhomogeneity, but at the same time increases the total magnetic flux inclosed by the current circuit. Expressions have been derived[3] for the magnitude of the current due to the effect, when the paths of the ions are long and when they are short. In the sun's atmosphere the ionic pressures are such that the conditions for long free paths are satisfied.[1] The value of the resulting current density (i_x) for long free paths, on the assumption that the magnetic field (H_z) is in the positive z-direction, is given by

$$i_x = -\frac{2NkT}{\pi H_z^2}\frac{dH_z}{dy}, \tag{1}$$

where N is the number of ions per cubic centimeter, k the Boltzmann constant, T the absolute temperature, and dH_z/dy the component of the magnetic gradient perpendicular to the direction of the impressed magnetic field. In addition to the requirement of long free paths, the value of (1) is conditional upon the existence of a boundary which is deep compared to the radius of the spiral generated by the ion as it spirals about the impressed magnetic field. This additional requirement has been met in the sun's atmosphere.

A complete study of the relation of the present effect to the stability of the system has not yet been made, and the exact mechanism by which an original "seed" disturbance is built up into a large magnetic spot is still imperfectly understood. Qualitatively, the process is somewhat as follows: An initial ionic vortex at the base of the spot sets up a small magnetic field as a result of the ionic motion and a slight centrifugal separation of the charges. This magnetic field has radial symmetry about an axis perpendicular to the surface of the sun and may be replaced in a first approximation by a small magnetic shell. This magnetic shell is effectively surrounded by a highly ionized conducting sheet in which may flow currents due to the inhomogeneity of the impressed "seed" field. The current tends to reduce the local impressed magnetic gradient, but in so doing increases the total flux inclosed by the current and the magnetic gradient outside the element of current. Consider the circular current flowing in the XY-plane due to thermal motion of the ions and

[1] R. Gunn, "The Sun's Radial Magnetic Gradient and Atmosphere," *ibid.*, **33**, 614, 1929.

the gradient produced by the magnetic shell placed at the origin in such a manner that the field outside the shell is in the positive Z-direction. The current will flow in such a direction as to reduce the impressed gradient, and by means of equation (1) we find that the magnetic shell due to the resulting current is in the same direction as the original shell. We thus have the definite possibility for a regenerative action such as occurs in a dynamo, since the flowing current increases the original field rather than decreases it. It is interesting to note that in regions outside the current element the gradient is increased, while inside it is decreased. Thus if the specific ionization is constant, the seat of currents would be expected to move outward or grow, while the field inside the current ring will tend to become uniform. Equilibrium conditions have not yet been completely worked out, but it is evident that if an explanation similar to the foregoing is to account satisfactorily for the magnetic field associated with sun-spots, then the computed currents derived from data that are approximately known must yield a magnetic field of the same order of magnitude as that observed.

We shall consider a typical circular spot to be made up of three regions, and shall select values which are representative of its size, temperature, and ionization. The umbra, which has a radius R_u, will be taken as a region of gaseous expansion wherein the temperature drops well below the surrounding temperatures to roughly 5,000° or less, and therefore the ionization may be entirely neglected in comparison with the surrounding region. Outside the umbra is the penumbra, whose outer radius approximates $2.5R_u$ and temperature 5,500°. This intermediate region is ionized, and currents and diamagnetism in it probably play an important part in the production of the magnetic field. A more interesting region, however, just outside the penumbra must be considered to be the seat of the largest currents. This "outer region" has an external radius R_0 of roughly $4R_u$ and has a mean temperature of 6,300°, so that we may assume that the product of ionization and temperature in this region is large compared to that of the umbra or penumbra. This high temperature, which is well above the mean temperature of the sun, may be attributed to the compression of the returning gas which expanded originally in the umbra. Moreover, since the ions spiral

about the magnetic field in such a way that their net mass motion is along the magnetic lines of force, we are led to assume that the magnetic flux from the umbra returns through the outer- or high-temperature region rather than through a companion spot. This conclusion is supported also by the observed fact that unipolar spots are nearly as numerous as bipolar groups.

We shall assume further that, owing to the diamagnetism of the ionized region and the magnetic field of the spot, a region of large radial magnetic gradient surrounds the penumbra which depends on the ionic density in the same manner and is of the same order of magnitude as the extrapolated value of the outward radial gradient of the sun's general magnetic field. Mount Wilson data indicate that the gradient of the general magnetic field which has been shown to be due to diamagnetism[4] is about 5×10^{-6} gauss/cm at an altitude of 250 km above the photosphere. The ionic pressure is known to decrease rapidly with the altitude, and it seems reasonable to assume that the mean gradient surrounding the sun-spot in regions near the low-lying seat of the current is one hundred or more times the observed value at higher levels. For the purpose of calculation, we shall therefore take the gradient as 5×10^{-4} gauss/cm.

The magnetic field at the center of a conducting ring carrying a current I having an internal radius R_i and external radius R_o is given by

$$H=\left(\frac{2\pi I}{R_o-R_i}\right)\log\frac{R_o+\left(R_o^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}{R_i+\left(R_i^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}, \tag{2}$$

where t is the length of the ring parallel to the axis of symmetry. In the present case t is the thickness of the conducting layer and R_o and R_i are the radii of the external and internal boundaries of the outer region of high temperature which is taken to be the seat of currents.

The total current flowing in the ring is, by aid of equation (1),

$$I=\frac{2NkT\,t\,(R_o-R_i)}{\pi H_z^2}\frac{dH_z}{dr}. \tag{3}$$

This, combined with equation (2), becomes

$$H=\frac{4NkT\,t}{H_z^2}\frac{dH_z}{dr}\log\frac{R_0+\left(R_0^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}{R_i+\left(R_i^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}. \tag{4}$$

We shall assume that the magnetic field at the seat of current H_z is proportional to the field at the center of the current ring, or $H=AH_z$, where A is a constant which does not depart from unity by more than an order of magnitude. For this preliminary investigation it will be arbitrarily taken as unity.

Previous investigations[1] have shown that long free-path phenomena of the type under consideration persist until the radius of the circle generated by the ion spiraling about the impressed magnetic field approximates the free paths of the ion. The molecular density corresponding to this critical limit is given by

$$N_{\text{critical}}=\frac{4eH_z}{\pi\sigma^2(2mkT)^{\frac{1}{2}}} \tag{5}$$

where σ is the diameter according to the kinetic theory of the molecule or ion and m the mass of the ion. On the sun

$$N_{\text{critical}}=1.2\times10^{15}H_z.$$

If we substitute this critical value of N in equation (4), it should lead to the correct value for the observed field, since N increases regularly as we approach the photosphere and the maximum currents will result when N approximates N_{critical}. Thus finally we have

$$H^2=\frac{16eA}{\pi\sigma^2}\left(\frac{kT}{2m}\right)^{\frac{1}{2}}t\frac{dH_z}{dr}\log_e\frac{R_0+\left(R_0^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}{R_i+\left(R_i^2+\frac{t^2}{4}\right)^{\frac{1}{2}}}. \tag{6}$$

The values of R_0 and R_i may be observed directly in the upper layers of the sun's atmosphere, but in the layers near the photosphere in

[1] *Loc. cit.*

which we are interested, the values can only be estimated. In any case their ratio will be substantially the same as the value actually observed, and we may without great error take $R_0 = 25 \times 10^8$ cm and $R_i = 16 \times 10^8$ cm. In a typical spot where R_u, the radius of the umbra, is 6×10^8 cm, the observed field is 2700 gauss. Putting in equation (6) the values appropriate to the sun, namely, $e = 1.59 \times 10^{-20}$ e.m.u., $\sigma = 2 \times 10^{-8}$ cm, $k = 1.37 \times 10^{-16}$, $T = 6300°$, $m = 9 \times 10^{-28}$ gm, $t = 5 \times 10^6$ cm, and $dH_z/dr = 5 \times 10^{-4}$ gauss/cm, then the computed value of H, the magnetic field at the center of the sun-spot, turns out to be 2010 gauss. This is considered to be in satisfactory agreement with observation since the actual values of A, t, and dH_z/dr are all imperfectly known.

The present theory seems to indicate that for sun-spots of moderate size the central field-strength is nearly independent of the diameter, while direct observation shows that the field is nearly a linear function of the diameter. The discrepancy may or may not have real significance, since measurements of the magnetic polarization of the sun-spots indicate that the present observations are confined largely to the outer surface of the sun-spot.

The foregoing considerations regarding the magnetic field of sun-spots are not complete and will doubtless require future modification. In its present form the theory appears to account adequately for the major features of the magnetic field without the introduction of large mechanical forces and mass rotation of the ions to account for the currents. The calculations which have just been made indicate that the ionic currents are adequate to produce the original field in the steady state, and strongly suggest that in a diamagnetic and highly ionized region the phenomena are regenerative and self-supporting.

NAVAL RESEARCH LABORATORY
WASHINGTON, D.C.
March 7, 1929

ENERGY SPECTRA OF THE STARS[1]

By C. G. ABBOT

ABSTRACT

A sensitive radiometer.—Fragments of *flies' wings* are used as *vanes*, and the exceptionally light *suspension* is sealed into a *fused-quartz tube*, optically figured opposite the vanes and mirror and *filled with hydrogen* at 0.23-mm pressure. The description includes various data regarding the weights, moments of inertia, advantages of hydrogen as compared with air, and sensitiveness of the apparatus.

Radiometric observations of stars.—The instrument was used to measure the distribution of energy in stellar spectra formed by a 60° flint-glass prism at the coudé focus of the Mount Wilson 100-inch telescope. The deflections were observed at a scale distance of 6 m by a special reading device, with a probable error of 2".8 in the rotation of the suspension. This figure includes errors of reading and those due to all other accidental causes. *Spectral energy-curves for eighteen stars and two planets were observed*, sometimes on two or three nights. The faintest star measured is of mag. 3.8. A comparison of the results with those obtained by the author in 1923 with a larger radiometer shows, in general, an agreement as good as could be expected from the accuracy of the observations.

In the year 1923, with a radiometer prepared for me by the late Dr. E. F. Nichols and his colleague, Dr. J. D. Tear, I measured relatively the intensities in spectra of the sun and nine very bright stars of various types, whose radiation was collected by the 100-inch telescope of the Mount Wilson Observatory.[2] Obviously it is desirable to measure a considerable number of specimens of each type of stellar spectrum, but, in order to do so, it appeared necessary to secure either greater sensitiveness or less accidental disturbance of the record.

The radiometer system employed in 1923 had vanes of mica, 0.5 mm wide by 1.5 mm high, separated 2.5 mm between centers, and the stem carried a mirror of fair dimensions, suitable to telescope and scale observation. It occurred to me that a very much lighter system might show increased sensitiveness. In 1926, I therefore constructed a system with vanes consisting of houseflies' wings separated by only 1 mm, and with a mirror of microscope cover-glass only 1 mm square. On trial, this system showed very high sensitiveness, but was so excessively damped at the air pressure best suited

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 380.

[2] *Mt. Wilson Contr.*, No. 280; *Astrophysical Journal*, 60, 87, 1924.

to the radiometer effect as to be quite useless. Dr. Anderson suggested that hydrogen might be an advantageous substitute for air.

Accordingly, in 1927, I made up two very light systems, one with 1.2 mm, the other with 2.0 mm between vane centers, each with vanes approximately 0.4 mm wide and 1.0 mm high. I made, also, bolometric strips which were used with them in the following manner. Three parallel test-tubes were joined by tubes, so that all three could be evacuated and filled together to any desired pressure with air or gas. In two tubes were sealed the radiometer systems, suspended on fine quartz fibers, and in the third were sealed

TABLE I

COMPARATIVE RESULTS, AUGUST 5, 1927

INSTRUMENT		SMALLER SYSTEM				LARGER SYSTEM				BOLOMETER
Pressure mm *Hg*	Gas	Time of First Swing, Seconds	Deflections of Successive Swings, mm			Time of First Swing, Seconds	Deflections of Successive Swings, mm			Deflection, mm
			1st	2d	3d		1st	2d	3d	
0.19	H_2	1.25	92	52	38	3.0	165	50	18	46.5
0.19	Air	1.6	82	22	8	4.0	110	0	0	61.3
0.26	H_2	1.3	98	45	31	3.1	125	35	7	36.8
0.26	Air	2.0	93	21	7	4.2(?)	110	0	0	60.4

the bolometer strips. Having been charged to a desired gas pressure, the instruments were exposed to a beam of light of nearly constant intensity. The deflection and the time of swing of each radiometer were observed when one of its vanes was illuminated. Also the deflection of a connected galvanometer was observed when one bolometer strip was exposed to the beam. By changing the pressure and the kind of gas, two series of measures were obtained in this manner, one for hydrogen, the other for air. These showed two kinds of results: from the bolometric deflections, the relative rise of temperature of blackened surfaces within the competing gases; and from the deflections of the suspended vanes, the comparative behavior of the two gases as regards radiometer effect and damping effect. Table I shows some of the results near optimum pressures.

For these light systems it thus appears that, although hydrogen

is somewhat less efficient than air as regards rise of temperature of the blackened surface, the loss of efficiency is not so great as to offset the advantage of the very decided decrease in damping. It seemed evident that if the quartz fiber of a hydrogen-surrounded radiometer should be thinned until the time of swing matched that of a similar air-surrounded radiometer of 10-second single swing, the deflection of the hydrogen-surrounded instrument would be much the greater. Subsequent experiments demonstrated that for a system of the same dimensions as the smaller one, the deflections in hydrogen at 0.23-mm pressure for single swings between 0.5–2.5 seconds were almost exactly proportional to the square of the time of swing. At 2.5 seconds, the vane was far from critical damping. This leads to the expectation that such a system would show a decided gain in deflections from 2.5 up to 10 seconds single swing—probably not sixteen fold, corresponding to the square of the time of swing, but perhaps tenfold.

Anticipating the success of hydrogen as a substitute for air, I had obtained, by the kindness of Director Burgess, of the Bureau of Standards, and the skill of Mr. Sperling, of the Bureau, a special pyrex glass tube with optically figured windows. In this tube Mr. A. Kramer, instrument-maker of the Smithsonian Astrophysical Observatory, had arranged a mechanism operated by a magnetic device from without. It was expected that when the tube had been filled to the proper hydrogen pressure and hermetically sealed, the vanes and mirror could be turned by magnetic influence to a proper orientation with respect to the windows. On trial, however, this device failed, because the rotation of the mechanism set up currents in the gas which so irregularly displaced the radiometer system that it was quite impossible to set it at a proper angle for observing.

In 1928, at the suggestion of Director W. S. Adams, of the Mount Wilson Observatory, a simpler plan was adopted. A fused-quartz tube, about 4 cm in diameter, was made to my order by the General Electric Company. In its middle section it was figured within and without to concentric circular curvature by Mr. Kinney, of the optical shop of the Observatory. This tube was exhausted to 0.0001-mm pressure through a liquid-air trap, washed out repeatedly with electrolytic hydrogen, filled to 0.23-mm pressure therewith, and

sealed up with the suspended radiometer inclosed. In this work, which had to be done on Mount Wilson, I was greatly assisted by Mr. Pompeo, whose skill in working quartz and glass is very great.

Since the quartz tube was optically figured, and of excellent clearness, it made no difference how the mirror and vanes were oriented. It was only necessary to support the tube from a ground-joint in a brass case in order that the whole tube containing the radiometer system might be rotated until the mirror looked out through the window of the brass case toward any desired part of the reading scale. A quartz window was employed opposite the vanes, so that, if desired, the experiments might be made with very short wave ultra-violet, or very long wave infra-red stellar radiation. In the experiments of 1928, however, a flint-glass prism was employed, and generally the spectral range was only from about 0.5 to about 2.5 microns.

A quartz fiber, so fine as to be handled with extreme difficulty, was used as a suspension. The vanes of the radiometer system were 0.4 mm wide and 1.0 mm tall, and but 1.2 mm between centers. Each vane had three parallel laminae of houseflies' wings, of which the front one was painted dead black, the two rear ones being unpainted. The laminae were separated by spaces of about 0.1 mm, in order that the communication of heat from front to back would be greatly impeded. The mirror, 0.9 by 1.0 mm and situated about 3 cm above the vanes, was made of microscope cover-glass which had been ground and polished to about half the usual thickness and platinized on both sides by sputtering. The whole system, including vanes, glass stem, and mirror, weighed 0.94 mg.

Some notion of the excessive fineness of the quartz fiber, 10 cm long, may be had from the fact that when suspended in full atmospheric air pressure, forty-four complete turns of the top support were made before the suspended system began to rotate in response. The air, in other words, acted as if viscous, like tar, although the vanes were so small and near together that very slight force must suffice to rotate them.

The following are details relating to weights and moments of inertia of the radiometer parts. The weight of 22 flies' wings, approximating 200 sq. mm in area, was 1.7 mg. After blackening, 8

flies' wings having an area of approximately 70 sq. mm weighed 3.1 mg. Hence the weights of the four fragments of unblackened flies' wings which were used were negligible compared to the two fragments of blackened wings. The moment of inertia of the fly-wing vanes as actually cut and mounted is 132×10^{-9} g. cm².

After grinding, polishing, and platinizing, the mirror glass weighed 0.20 mg per square millimeter. The moment of inertia of the mirror used, 0.9×1.0 mm, is 121×10^{-9} g. cm². The glass cross-rod weighs 0.023 mg, and its moment of inertia is 37×10^{-9} g. cm². Compared to these, the moments of inertia of the vertical glass stem and wax used are negligible.

Hence, the total amount of inertia is 290×10^{-9} g. cm², nearly equally divided between the vanes and the non-deflective parts.

The total weights are as follows:

Vanes	0.035 mg
Mirror	.180
Cross-rod	.023
Vertical	0.700
Total	0.938

The glass parts were cemented together with minute quantities of shellac from which all alcohol had been evaporated by heat and reduced pressure. The vanes and mirror were cemented on with minute quantities of beeswax. Since there was some doubt as to whether beeswax would evaporate sufficiently to contaminate the hydrogen within a moderate time, an area of 1400 sq. mm of mica was coated with beeswax on August 24, 1926, and weighed after standing a half-hour under a bell-jar with phosphorus pentoxide. After a similar drying on October 30, the weight appeared unchanged, and on June 21, 1927, there appeared to be a loss of weight of 0.2 mg. Throughout this period the temperature of the beeswax considerably exceeded that to which it would be subjected on Mount Wilson. When the times, areas, and temperatures involved are taken into consideration, the evaporation appears to be negligible, even compared to the weight of hydrogen in the containing case.

During the exhaustion and filling of the quartz tube, rough tests indicated that the system would have a single swing of about 12 seconds. But when the tube was inserted in its brass case it was as-

tonishing to find that the time of single swing was only 0.5 second. When the apparatus was left over night, the time of swing increased to nearly 2 seconds. After a week's rest it appeared again to be above 10 seconds. It was then found necessary, however, to remove the tube to clean finger-marks from the quartz surfaces. On reassembling, it was found that the time of single swing had diminished to 1 second. On the assumption that electrical charge, due to friction, was the cause of the controlling field, sufficient water was introduced to saturate the air of the outer case. The apparatus was untouched for more than a week afterward before trying it for the first time on stellar spectra. But the time of swing rose only to 1.5

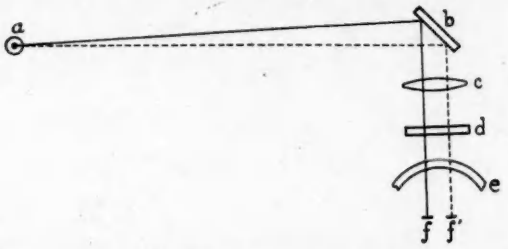

FIG. 1.—Set-up for test of sensitiveness: *a*, candle; *b*, silvered mirror; *c*, glass lens; *d*, quartz plate; *e*, quartz tube; *ff'*, radiometric vanes.

seconds. After a further interval of nearly three weeks before a second trial, the time of swing remained 1.5 seconds. I still think, however, that the cause of this unduly quick action is electrostatic, and hope that before another trial is made the instrument will recover its longer time of swing and high sensitiveness.

The following test, with the set-up shown in Figure 1, was made to measure the sensitiveness of the radiometer as actually used with a single swing of 1.5 seconds. The radiometer was exposed to rays from a candle, *a*, at 2.4 m distance, reflected at 45° by a silvered glass mirror, *b*, through a crown-glass lens, *c*, of 3-mm thickness and 3.7-sq. mm aperture, and thence through two quartz plates, *d*, each of 3-mm thickness, to focus on a radiometer vane, *ff'*. Since this was only about two-thirds as tall as the candle-flame image, there was a large loss of radiation. Under these circumstances a deflection of 80 mm was produced on a scale at 40 cm distance when the image of the flame was shifted from one vane to the other, just as in

actual practice with stellar spectra. During observations on stars, the scale distance was 6 m instead of 40 cm, thus giving fifteen fold greater sensitiveness than during the tests. Had the desired time of single swing of 10 seconds or more been available, the sensitiveness would, I believe, have been further increased nearly twenty fold. A rough test made with the candle source on the only day when the

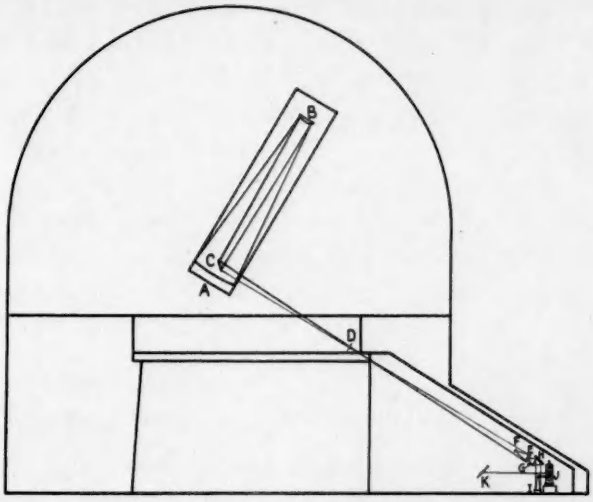

FIG. 2.—Diagram of stellar spectral set-up

time of swing exceeded 10 seconds showed a deflection on the scale at 40 cm which was evidently several times as great as the entire length of the 200-mm scale. Whether this increased sensitiveness would have produced a very great gain in accuracy of the results I am not prepared to say, for no doubt the accidental errors of reading would have increased with a longer time of swing and greater sensitiveness.

The optical system employed is shown diagrammatically in Figures 2 and 3. Rays reflected by the 100-inch telescope mirror *A* came nearly to a focus at the top of the telescope tube. Thence they were reflected down the tube by a convex mirror *B*, and thence by a plane mirror *C*, southward down the hollow telescope axis converging toward the coudé focus at *D*. At that focus was placed a circular variable aperture, about 3 mm in diameter at maximum, through which the rays diverged for a distance of about 16 m to fill a con-

cave mirror E, of 50-cm aperture and 1-m principal focus, situated near the floor of the constant-temperature room. Thence they were reflected back to a 10-cm convex mirror F, near the conjugate focus. By this small mirror the rays were rendered parallel and reflected southward and downward through a hole in the center of the 50-cm mirror to pass, at approximately minimum deviation, through a 60° flint-glass prism G, with faces 13 cm wide and 17 cm long,

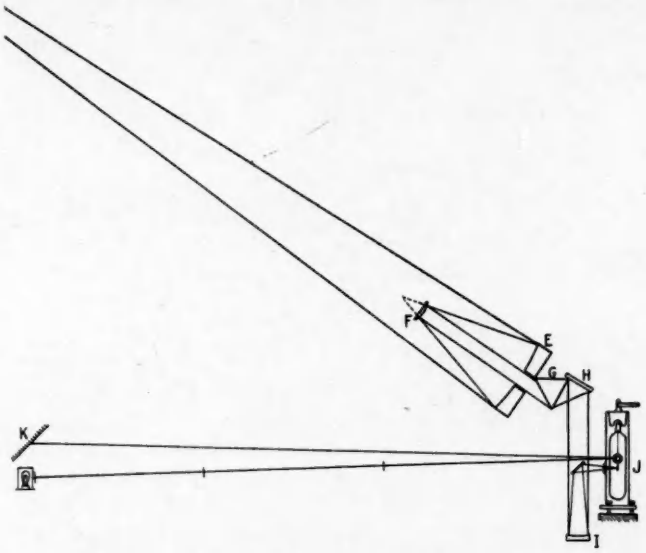

FIG. 3.—Means of forming the spectrum and of observing deflections

whose refracting edge lay horizontally. Beyond this and attached to the prism, was a plane mirror H, which reflected the rays nearly vertically downward to a concave mirror I of 45-cm focus. From this mirror they were reflected vertically upward to a small plane mirror near the focus, and thence horizontally southward to the focus of the spectrum on the vane of the radiometer at J.

With the arrangement described, the spectrum was vertical. In order to make readings at different points in the spectrum, means were provided for rotating the prism and its attached plane mirror about a horizontal axis. Whenever the 3-mm circular diaphragm at the coudé focus of the great telescope was filled with light, the width of the spectrum at the radiometer was about 0.4 mm and

equal to the width of the radiometer vanes. For many of the stellar observations, the diameters of the star images were so small that the spectral width observed hardly exceeded half the width of the radiometer vanes. With Mars and Jupiter, the images were so large that only a small central part of their rays passed through the diaphragm. The vertical extension of the spectrum was about 30 mm, between wave-lengths 0.48 and 2.00 microns. Thus, for any single setting, the radiometer vanes 1 mm high subtended about one-thirtieth of the average extension of the observed spectrum. Means were provided for horizontally displacing the spectrum at pleasure, from one vane to the other, and the deflections observed in the stellar spectra all corresponded to such displacements.

It is unfortunate that the rays suffered losses of intensity at so many reflecting surfaces, including eight silvered mirrors, two prism faces, and the four faces of the two quartz plates traversed by the beam. These losses differed, of course, for different wave-lengths. For the yellow rays the total loss may have been approximately 75 per cent, and for green or blue rays was still more. Owing to the impossibility of making these delicate stellar-spectrum energy observations with the radiometer, except under conditions of great freedom from vibration and of nearly constant temperature, it is difficult to see how these large losses of energy could have been much reduced. Were it possible to observe the energy of stellar spectra with the thermopile near the direct focus of the telescope mirror, as Pettit and Nicholson observe total stellar radiation, about half of these losses could be avoided.

The success of the experiments depended on the means used for observing the deflections. These are indicated by Figures 3 and 4. A 100-watt incandescent lamp with ring-shaped filament sent a tiny ray through several diaphragms and through a long-focus lens to illuminate the mirror of the radiometer, about 5 m distant. Owing to the shape of the constant-temperature room, the beam, both in approaching and receding from the radiometer, had to be turned through 90° by small mirrors. The spot of light fell upon a long horizontal cylindric lens, 6 m distant from the radiometer, and beyond it, upon a ground glass, not marked in any way. A fairly bright blur of light, nearly circular in form and about 1 cm in di-

ameter, was thus produced. Behind the ground glass was a little movable frame, operated at pleasure by a screw or by hand displacement, which carried opposite the spot of light a bit of cork cut so that when placed centrally it obscured all but a thin halo of light. Below the ground glass was a scale of equal parts ruled on the ground-glass base-side of a long 45° prism of glass, which could be illuminated from beneath by total reflection at the hypotenuse of the prism. A cross-wire, carried by the movable frame, indicated on the illuminated scale the number of whole turns of the screw corresponding to a given setting. A divided head of ground glass which could also be illuminated indicated the hundredths of a turn of the

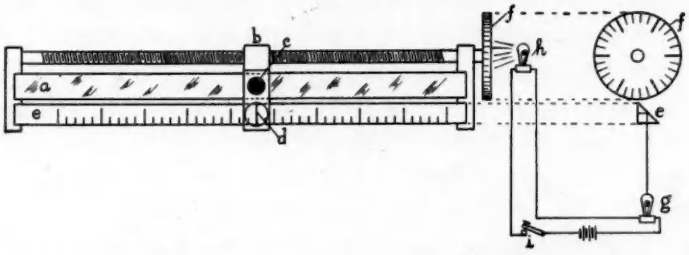

FIG. 4.—Special scale for reading deflections: *a*, plain ground glass; *b*, sliding frame; *c*, cork occulter; *d*, cross-hair; *e*, triangular glass scale of screw divisions; *f*, ground-glass divided screw-head; *g*, *h*, lamps for scales; *i*, switch.

screw. As the pitch of the screw was 3 mm, the settings could readily be observed to 0.03 mm, and by estimation of tenths, to 0.003 mm, if desired.

The method of reading deflections will now be apparent. By a photometric comparison of the total light on the right and left of the occulting cork, the observer, working in almost total darkness, made the setting. Then by pressing a button the scales were momentarily illuminated and the accurate readings were made. As will appear from the tabular values, the average probable error of a single reading was only 0.06 mm, including not only the errors of a single setting but also those displacements produced by changes of transparency of the air, "boiling" of the star image, and unsteadiness of the radiometer.

Director W. S. Adams very greatly aided me on all three nights of observation by finding the stars at the coudé focus and recording

all observations. Night assistants Bebee and Nelson kept the stars accurately in the field, and to them also my thanks are due.

Table II gives a list of the objects, in order of their spectral type, observed in the year 1928, and notes on the conditions of observation. Figure 5 shows curves representing the energy distributions found in the prismatic spectrum. For convenience in plotting, a somewhat different order of arrangement from that of the table is used in the figure.

TABLE II

CELESTIAL OBJECTS OBSERVED IN 1928

Object	Mag.	Sp.	Dates Obsd.*			Spectral Range	Results
						microns	
β Orionis	0.3	B8p	....	....	13	0.423–0.905	Good
α Lyrae	0.1	Ao	25	26	13	.437–1.751	Unsatisfactory
α Cygni	1.3	A2p	25	....	13	.472–1.316	Good
α Aquilae	0.9	A5	....	26	....	.437–1.316	Good
α Canis Min.	0.5	F5	....	....	13	.472–1.316	Excellent
α Persei	1.9	F5p	....	26	13	.520–1.751	Good
γ Cygni	2.3	F8p	....	....	13	.472–2.224	Fair
α Aurigae	0.2	Go	25	....	....	.472–2.224	Good
Mars	.....	.....	....	....	13	.520–1.751	Good
Jupiter	.....	.....	....	26	....	.520–1.751	Good
β Ceti	2.2	Ko	25	26	....	.589–1.751	Good
γ Aquilae	2.8	Ko	....	26	....	.589–2.224	Excellent
α Boötis	0.2	Ko	25	....	....	.589–2.224	Good
α Tauri	1.1	K5	25	....	....	.589–2.224	Good
α Orionis	0.9	Ma	25	....	....	.589–2.224	Excellent
β Andromedae	2.4	Ma	25	26	....	.589–2.224	Good
β Pegasi	2.6	Ma	25	....	....	.589–2.224	Excellent
δ Sagittae	3.8	Map	25	26	....	.589–2.224	Unsatisfactory
α Herculis	Var.	Mbp	25	26	....	.589–2.224	Excellent
o Ceti	Var.	Md	25	26	13	0.589–2.224	Good

* The entries refer to August 25, 26, and September 13, respectively.

As an example of the direct observations, Table III shows the complete set of measurements made on β Orionis, September 13, 1928. The places in the spectrum are recorded in turns of the tangent screw which rotates the prism from the position of the yellow sodium lines. The readings of the radiometer (marked R) are given in turns and hundredths of a turn of the screw of the observing device. The corresponding deflections (marked Δ), which, when positive, produced smaller scale readings, are computed by subtracting the intermediate reading of each successive triad from the mean of the values preceding and following it.

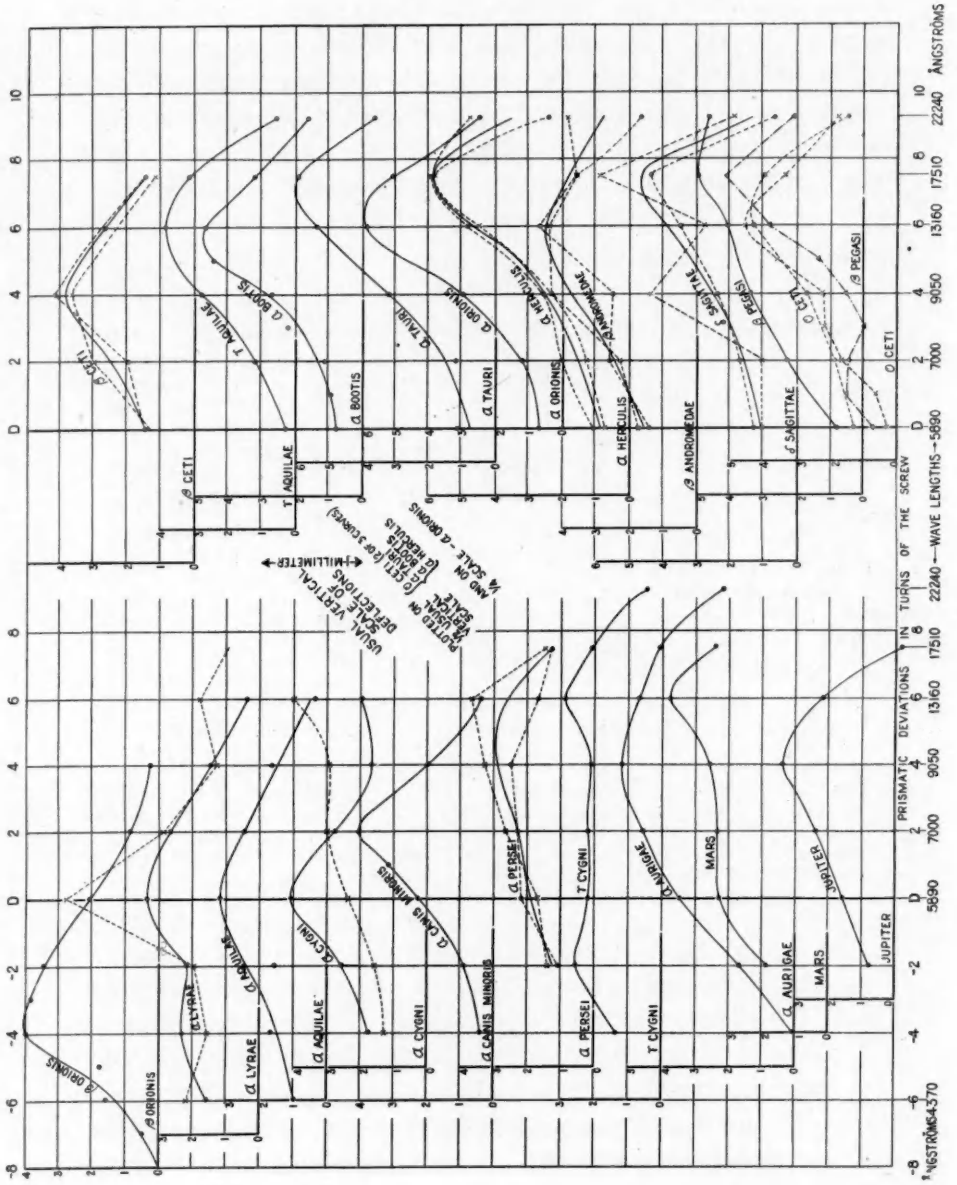
PRISMATIC DEVIATIONS IN TURNS OF THE SCREW
22240—WAVE LENGTHS—5690
ANGSTRÖMS
ANGSTRÖMS 7370
USUAL VERTICAL SCALE OF DEFLECTIONS
PLOTTED ON USUAL VERTICAL SCALE
SCALE - α ORIONIS
α CETI
T AQUILAE
α BOOTIS
α TAURI
α ORIONIS
α HERCULIS
β ANDROMEDAE
ζ SAGITTAE
β PEGASI
O CETI
α LYRAE
α AQUILAE
α CYGNI
α CANIS MINORIS
α PERSEI
T CYGNI
α AURIGAE
MARS
JUPITER

FIG. 5

From Table III, which is in no respect exceptional, it appears that the drift, as usual, was toward smaller readings, but amounted to only about 0.3 division, or 1 mm, in approximately 20 minutes.

TABLE III

OBSERVATIONS OF β ORIONIS, SEPTEMBER 13, 1928

Time of Start, P.S.T.	15^h48^m		15^h32^m		15^h52^m		15^h34^m		15^h54^m	
Place in Spectrum	−7		−6		−5		−4		−3	
Wave-Length, Microns	0.423		0.437		0.454		0.472		0.494	
Reading, Deflection	*R*	Δ	*R*	Δ	*R*	Δ	*R*	Δ	*R*	Δ
Energy readings in 60° glass prismatic spectrum, expressed in scale division of 3 mm and fractions thereof	20.00		20.35		19.91		20.34		19.99	
	19.88	0.08	.23	0.11	.71	0.20	19.91	0.39	.65	0.30
	.92		.34		.91		20.26		.91	
	.86	.00	.17	.14	.74	.16	19.92	.42	.46	.42
	.80		.28		.90		20.42		.85	
	.80	0.06	.00	0.22	.69	0.19	19.93	0.43	.44	0.44
	19.91		20.16		19.86		20.29		19.92	
Mean	.....	0.05	.....	0.16	.....	0.18	.....	0.41	.....	0.39
Probable error	.....	0.018	.....	0.026	.....	0.010	.....	0.010	.....	0.035

Time of start, P.S.T.	15^h37^m		15^h40^m		15^h42^m		15^h44^m	
Place in Spectrum	−2		0		+2		+4	
Wave-Length, Microns	0.520		0.589		0.700		0.905	
Reading, Deflection	*R*	Δ	*R*	Δ	*R*	Δ	*R*	Δ
Energy readings in 60° glass prismatic spectrum, expressed in scale division of 3 mm and fractions thereof	20.30		20.11		20.17		20.30	
	20.00	0.28	19.91	0.20	.17	0.07	.31	−0.03
	20.25		20.11		.31		.25	
	19.90	.38	.00	.16	.24	.10	.13	+0.08
	20.30		20.20		.37		.17	
	19.86	0.40	19.94	0.27	.23	0.11	.10	+0.04
	20.21		20.22		20.30		20.12	
Mean	.....	0.35	.....	0.21	.....	0.09	.....	0.03
Probable error	.....	0.038	.....	0.025	.....	0.010	.....	0.025

Also, since the mean probable error of a determination (the mean of three complete measurements) is 0.021 divisions, or 0.063 mm, and, since this rests on seven readings of the scales, the mean probable error of a single reading is $0.063\sqrt{7}=0.17$ mm. This figure of demerit is swelled by all the causes which tend to produce accidental

deflections, as well as by the errors of reading itself. It corresponds to a rotation of the radiometer mirror through 2″.8.

The maximum deflection in the spectrum on this occasion was only 0.41 division, or 1.23 mm, yet the energy-curve is rather well delineated and smoothly indicated.

With Rigel omitted, which has already been given in detail, Table IV includes a summary of the observations made in 1928.

Times and air masses are not given more exactly in Table IV because, when corrections to a mean air mass of 1.30 were computed, corresponding to the mean atmospheric transmission coefficients for Mount Wilson, it proved that only in the case of Procyon were they large enough to affect the results.

In the stellar radiometer observations of 1923, the selective transmissions of the apparatus and the atmosphere were allowed for in order to give the spectral distribution as in free space. For this purpose the energy-curve of the sun's spectrum, as observed with the stellar set-up of telescope and radiometer, was compared with that determined by bolometric work in 1920 and 1922 on Mount Wilson.[1] As time did not permit a repetition of this comparison in 1928, the coefficients of transmission adopted in 1923 were made available by comparing prismatic energy-curves of 1928 with those of 1923 for all stars observed in both years. From this comparison it appeared that the infra-red portion of the spectrum, compared to the visible, was less intense as observed in 1928 than in 1923. This difference may have been due to an increased infra-red transparency of the lamp-black-coated fly-wing vanes used in 1928, as compared to the lamp-black-coated purled mica vanes, prepared by Dr. Tear and used in 1923. The results of the comparison are, on the whole, very accordant. (The curious infra-red observations of Rigel made in 1923 were omitted.) Unfortunately, data for rays of wave-length less than 0.589 microns are almost lacking. In extrapolating the curve, it seemed reasonable to assume only a slight change in that region. Table V gives the mean observed reduction factors; the adopted smoothed factors employed to reduce prismatic observations of 1928 to the conditions as of 1923; the factors determined in 1923 to reduce to the wave-length scale outside the

[1] *Smithsonian Miscellaneous Collections*, **74**, No. 7, 1923.

TABLE IV

The Mean Prismatic Deflections and Probable Errors, Observations of 1928

Object	Date P.S.T.*	Air Mass	Places in Prismatic Spectrum and Wave-Lengths, Microns																	
			−6 0.437		−4 0.472		−2 0.520		0 0.589		+2 0.700		+4 0.905		+6 1.316		+7½ 1.751		+9½ 2.224	
			Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.	Mean	P.E.
α Lyrae	26d 8h45m	1.01	0.22	0.03	0.16	0.03	0.20	0.02	0.58	0.03	0.29	0.07	0.13	0.09	0.18	0.02	0.10	0.01	.....	.....
	13 10 04	1.25	.16	.03	.23	.04	.22	.02	.34	.01	.27	.01	.15	.02	.04	.01	.....	.....	.....	.....
α Cygni	25 10 36	1.02	.....	.....	.18	.04	.26	.08	.41	.03	.28	.05	.17	.04	.20	.04	.....	.....	.....	.....
	13 13 27	1.56	.....	.....	.13	.02	.16	.04	.24	.01	.30	.02	.29	.00	.40	.01	.....	.....	.....	.....
α Aquilae	26 10 56	1.20	0.10	0.03	.17	.05	.16	.04	.32	.03	.25	.02	.17	.01	.04	.02	.....	.....	.....	.....
α Canis Min.†	13 16 16	2.44	.....	.....	.04	.04	.09	.01	.23	.03	.41	.04	.20	.05	.04	.05	.....	.....	.....	.....
	.....	.....	.....	.....	(.05)	.....	(.10)	.....	(.25)	.....	(.44)	.....	(.21)	.....	(.04)	.....	.....	.....	.....	.....
α Persei	26 15 44	1.07	.....	.....	.....	.....	.14	.02	.17	.04	.27	.05	.33	.01	.37	.01	.15	.02	.....	.....
	13 14 42	1.06	.....	.....	.....	.....	.11	.01	.22	.02	.23	.02	.25	.01	.17	.03	.13	.05	.....	.....
γ Cygni	13 12 00	1.21	.....	.....	.14	.04	.26	.02	.22	.03	.22	.03	.21	.03	.29	.05	.21	.04	0.05	0.02
α Aurigae	25 15 47	1.16	.....	.....	0.01	0.02	.17	.01	.35	.07	.46	.02	.52	.01	.47	.01	.41	.08	.22	.06
Mars	13 15 12	1.46	.....	.....	.....	.....	.19	.03	.33	.01	.34	.03	.36	.01	.48	.01	.14	.02	.....	.....
Jupiter	26 14 20	1.18	.....	.....	.....	.....	0.08	0.05	.16	.01	.24	.01	.34	.02	.22	.01	−.02	.04	.....	.....
β Ceti	25 13 26	1.71	.....	.....	.....	.....	.....	.....	.14	.02	.30	.05	.37	.05	.....	.....	.12	.04	.....	.....
	26 13 45	1.67	.....	.....	.....	.....	.....	.....	.15	.05	.20	.04	.42	.04	.27	.04	.15	.02	.....	.....
γ Aquilae	26 9 36	1.09	.....	.....	.....	.....	.....	.....	.03	.02	.11	.05	.28	.02	.39	.07	.32	.04	.06	.02
α Boötis	25 7 40	1.40	.....	.....	.....	.....	.....	.....	.18	.04	.23	.02	.53	.00	0.94	.08	0.63	.01	.33	.01
α Tauri	25 15 16	1.45	.....	.....	.....	.....	.....	.....	.23	.04	.24	.04	0.65	.03	1.06	.02	1.19	.03	0.73	.05
α Orionis	25 16 17	1.71	.....	.....	.....	.....	.....	.....	.28	.05	.49	.02	1.19	.05	2.35	.09	2.06	.09	1.01	.03
β Andromedae	25 13 58	1.02	.....	.....	.....	.....	.....	.....	.17	.06	.23	.07	0.44	.06	0.44	.10	0.36	.04	0.38	.02
	26 13 05	1.06	.....	.....	.....	.....	.....	.....	.14	.08	.26	.02	.25	.06	.47	.06	.36	.05	.17	.01
β Pegasi	25 12 53	1.01	.....	.....	.....	.....	.....	.....	.08	.13	.22	.14	.34	.14	.41	.07	.50	.03	.47	.05
δ Sagittae	25 9 53	1.05	.....	.....	.....	.....	.....	.....	.10	.06	.10	.05	.44	.02	.28	.02	.59	.07	.19	.10
	26 10 20	1.07	.....	.....	.....	.....	.....	.....	.13	.05	.17	.02	.24	.02	.35	.03	0.44	.03	.07	.03
α Herculis	25 8 28	1.15	.....	.....	.....	.....	.....	.....	.21	.02	.41	.05	.52	.05	.97	.02	1.20	.07	.49	.02
	26 7 56	1.10	.....	.....	.....	.....	.....	.....	.15	.02	.26	.02	.47	.05	.90	.07	1.18	.06	.96	.03
ο Ceti	25 14 37	1.35	.....	.....	.....	.....	.....	.....	.15	.05	.29	.01	.31	.04	.77	.05	1.03	.01	.63	.04
	26 14 59	1.30	.....	.....	.....	.....	.....	.....	.06	.01	.34	.02	.45	.02	.92	.03	0.67	.04	.36	.02
	13 14 06	1.28	.....	.....	.....	.....	.....	.....	.13	0.03	.19	0.01	.27	0.06	.43	0.02	.40	0.02	.15	0.01
	13‡14 06	1.28	.....	.....	.....	.....	.....	.....	0.26	.....	0.38	.....	0.54	.....	0.86	.....	0.80	.....	0.30	.....

* The days refer to August 25, 26, and September 13, 1928, respectively.

† Values given for α Canis Minoris in parentheses are corrected to air mass 1.3.

‡ ο Ceti appears to have been only half as bright on September 13 as on August 25 and 26. For easy comparison, the September values are doubled and are thus plotted in Figs. 5 and 6.

apparatus and the atmosphere; and the combined values of both factors.

Multiplying the values in Table IV by the combined factor in Table V, we obtain values for points on the normal scale outside the earth's atmosphere as given in Table VI.

The results given in Table VI are plotted in Figure 6. Most of the curves are fairly smooth and reasonable. As for individual discrepancies, it is obvious that the first three points on the curves for

TABLE V

REDUCING FACTORS

PLACE	FACTOR OF 1928		FACTOR OF 1923	COMBINED
	Obs.	Used		
−7	.....	0.44	2300	1010
−6	.....	.44	1400	616
−5	.....	.45	910	410
−4	0.45	.45	620	279
−3	.42	.46	460	212
−2	.....	.50	335	167
−1	.....	.53	253	134
0	0.59	0.59	188	111
2	1.03	1.03	96	99
4	1.71	1.71	41	70
6	1.74	1.74	25	43
7½	1.37	1.37	21	29
9¼	0.69	0.69	25	17

Rigel are too small, and that the observations of Vega, Deneb, and δ Sagittae are too irregular to give a fair indication of the march of their spectral energy-curves. A low value occurs at 0.589 microns in the spectrum of Capella, and surprisingly high values are found on all three nights at 0.700 microns in the spectrum of Mira.

It is interesting to compare the results with those of the year 1923. For this purpose the results of 1928 were reduced by applying appropriate factors, as nearly as possible, to the scale of those given in Table V of *Mount Wilson Contr.*, No. 280.[1] When this had been done, 43 places in the spectra of 8 stars were found for which ratios of intensities could be computed. Of these, 7 are so wild as to be rejected. The remaining 36 show an average deviation, 1928

[1] *Astrophysical Journal*, **60**, 87, 1924.

from 1923, of 19.8 per cent. Assuming that the observations of 1923 and 1928 are of equal accuracy, we find the probable error of a single average observation to be $(0.84 \times 19.8)/1.41 = 11.8$ per cent,

TABLE VI

STELLAR ENERGY SPECTRUM DISTRIBUTION; NORMAL SCALE, OUTSIDE THE ATMOSPHERE

OBJECT	DATE	PLACE ON PRISMATIC SCALE AND WAVE-LENGTHS, MICRONS								
		−6 0.437	−4 0.472	−2 0.520	0 0.589	+2 0.700	+4 0.905	+6 1.316	+7½ 1.751	+9½ 2.224
β Orionis*	13†	990	1140	584	233	89	21	.....	.....	.....
α Lyrae	26	1355	446	334	644	287	91	77	29	.....
	13	990	642	367	377	267	105	17	.....	.....
α Cygni	25	.....	502	434	455	277	121	86	.....	.....
	13	.....	363	267	266	297	206	172	.....	.....
α Aquilae	26	616	474	267	355	247	121	17	.....	.....
α Canis Min.	13	.....	139	167	277	436	149	17	.....	.....
α Persei	26	.....	.....	234	189	267	231	159	43	.....
	13	.....	.....	184	244	228	177	73	38	.....
γ Cygni	13	.....	390	434	244	208	149	125	60	8
α Aurigae	25	.....	28	284	388	455	369	202	119	37
Mars	13	.....	.....	317	366	337	256	206	41	.....
Jupiter	26	.....	.....	134	178	238	241	95	6	.....
β Ceti	25	.....	.....	.....	155	297	263	.....	35	.....
	26	.....	.....	.....	166	198	298	116	43	.....
γ Aquilae	26	.....	.....	.....	33	109	199	168	93	10
α Boötis	25	.....	.....	.....	200	228	376	404	183	56
α Tauri	25	.....	.....	.....	255	238	461	456	345	124
α Orionis	25	.....	.....	.....	311	485	844	1010	597	172
β Andromedae	25	.....	.....	.....	189	228	312	189	104	65
	26	.....	.....	.....	155	257	177	202	104	29
β Pegasi	25	.....	.....	.....	89	218	241	176	145	80
δ Sagittae	25	.....	.....	.....	111	99	312	120	171	32
	26	.....	.....	.....	144	168	170	150	128	12
α Herculis	25	.....	.....	.....	233	406	369	417	348	83
	26	.....	.....	.....	166	257	334	387	342	163
o Ceti	25	.....	.....	.....	166	287	220	331	299	107
	26	.....	.....	.....	67	337	319	396	194	61
	13	.....	.....	.....	144	188	192	185	116	26
	13‡	.....	.....	.....	288	376	383	370	232	51

* Additional observations for β Orionis: −7, 0.423 μ, 505; −5, 0.454 μ, 738; −3, 0.494 μ, 827.

† The dates refer to August 25, 26, and September 13, 1928, respectively.

‡ Values of preceding line doubled.

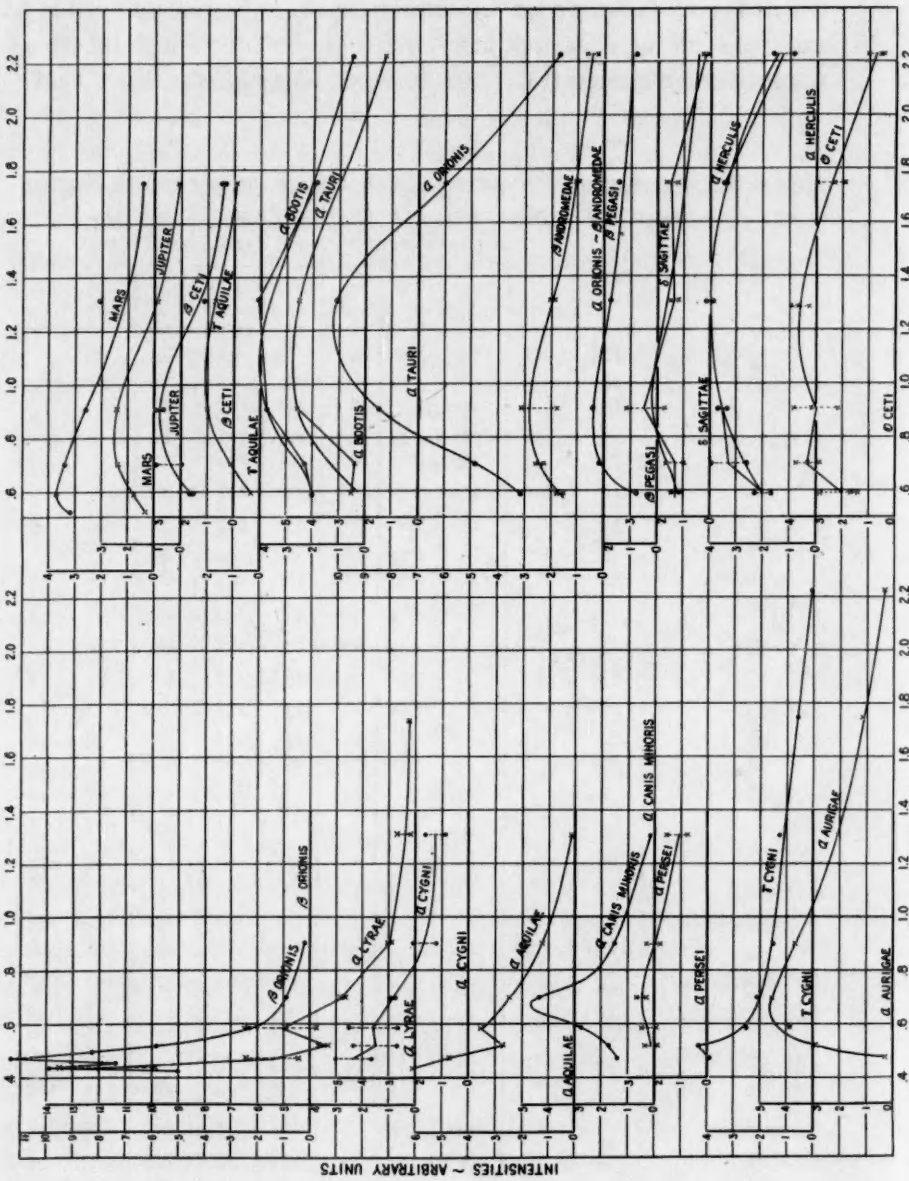
INTENSITIES ~ ARBITRARY UNITS
WAVE LENGTH ~ MICRONS
MARS
JUPITER
β CETI
γ AQUILAE
β CETI
α BOOTIS
α TAURI
α ORIONIS
β ANDROMEDAE
α ORIONIS ~ α ANDROMEDAE
β PEGASI
β SAGITTAE
α HERCULIS
α CETI
β ORIONIS
α CYGNI
α LYRAE
α AQUILAE
α CANIS MINORIS
α PERSEI
γ CYGNI
α AURIGAE

FIG. 6

which agrees fairly well with what is shown by Table III; for the average deflection in Table III is 0.21 divisions, and the average probable error is 0.021 divisions, or 10 per cent of the average deflection.

As a more concrete illustration of the degree of consonance between the observations of 1923 and 1928, consider the following table, where the results for α Orionis and β Pegasi for the two years are reduced to approximately the same scale.

TABLE VII

VALUES OF 1923 AND 1928 COMPARED

Place		0	+2	+4	+6	+7½	+9¼
Wave-length, μ		0.589	0.700	0.905	1.316	1.751	2.224
α Orionis	1923	378	614	1132	1202	918	407
	1928	407	635	1105	1440	782	225
	Δ	+29	+21	−27	+238	−136	−182
β Pegasi	1923	75	144	205	167	132	72
	1928	76	185	205	150	123	68
	Δ	+1	+41	0	−17	−9	−4

Although the agreement between the observations of 1923 and 1928 is as good as the accuracy of the results warrants us to expect, much improvement is needed before the spectral energy-curves of any of the stars can be regarded as well determined. If possible, continuous registration should be introduced in order that special features occurring within narrow ranges of wave-length may be detected. The measurements should be extended toward both shorter and longer wave-lengths than can penetrate the great glass prism used for these observations. More exact standardization of the observations in terms of radiation of known spectral energy distribution must be sought. It is highly desirable to obtain as much as tenfold greater sensitiveness than was available in 1928, without increased accidental disturbance, so that stars of the third magnitude may easily be observed. With such improvements a long campaign of observations of stellar spectral distribution would be justified.

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WAVE LENGTH ~ MICRONS

FIG. 6

WAVE LENGTH ~ MICRONS

REVIEWS

Einstein's Theory of Relativity. By MAX BORN; trans. HENRY L. BROSE. New York: E. P. Dutton & Co., 1922. Pp. 290. $5.00.

Relativity for Physics Students. By G. B. JEFFERY. New York: E. P. Dutton & Co., 1924. Pp. 150. $2.40.

Relativity and Gravitation. By T. PERCY NUNN. London: University of London Press, 1923. Pp. 162. $5.00.

Professor Born's book presents, within the technical limits set for it, a thorough welding of the main physical ideas of relativity and their antecedents in pre-relativity physics. It succeeds in preserving a rare balance between the points of view of the experimental and the speculative methods. The author takes pains to make clear his position in regard to the "physical reality" of admissible concepts. As an example we cite a passage on page 189 where, in discussing the mooted question of the aether, Professor Born remarks ". . . . Theorists may use their own judgment in equipping a vacuum with phase quantities (denoting state), fields, or similar things, with the one restriction that these quantities serve to bring changes observed with respect to material things into clear and concise relationship." Expressions such as this prepare the way gradually for the abstract character of much modern theory.

The fact is that the main emphasis of this book falls on the process of historical amalgamation referred to above. The selection of material from the fields of mechanics, optics, electrodynamics, and the theory of electrons is such as to lead the reader logically and inevitably to just those rifts in the foundations that necessitated the reformulation of principles embodied in the special and general theories of relativity. In this treatment the physical and pictorial sides stand out with unusual clearness. Professor Born has, however, set himself the extraordinarily difficult task of presenting such theories as those of Maxwell and Einstein without using the mathematical theory of limits, or, for that matter, the elementary trigonometric functions. The result is that he is forced to advise the reader to consider Maxwell's field equations as "mnemomics." This procedure is very bold in its application to the general theory of relativity, where it is necessary to pass from a finite geometry to a differential one. It may be questioned whether the book presents to the non-mathe-

matical reader to whom it is addressed more than a series of excellently drawn but nevertheless fragmentary sketches. For the slightly more advanced reader it is an invaluable approach to the theory of relativity, and should be read by anyone interested in that theory.

Professor Jeffery's little book, which derives its dynamic style from the lecture platform, carries us at one sitting through the last century of physics, leaving us well landed, at least, on the terrain of relativity. During the first third of the book, which is historical in character, no details or proofs retard the exposition. The second third introduces the special theory of relativity. Perhaps the outstanding feature of this section is the demonstration that the Maxwell equations of the electromagnetic field retain their form under a Lorentz transformation of variables. The inclusion of this point is particularly fortunate, due to its historic importance in the development of the theory. The last third takes up the general theory, with particular emphasis on the equivalence hypothesis. The theory of tensors is disposed of in five pages; the intentions of the author are confined to formal introductions. In the same spirit Einstein's field equations of gravitation are entirely omitted, the breach being partially filled by Schwarzschild's solution of them for the case of an isolated mass-point. This leads to a calculation of the advance of the perihelion point of a planetary orbit. Instead of deducing the deflection of light from the equations of motion of a planet by passing to the limit $ds=0$, Professor Jeffery makes the assumption (p. 107) that the same result can be obtained from a uniform gravitational field, and by an appeal to the principle of equivalence is thus able to derive the usual formula. By the same method he finds the formula for the shift of the spectral lines of a star. While this reasoning, as the author carefully points out, is by no means conclusive, yet it is very instructive from the physical point of view, and adds to the merit of the book.

Professor Nunn's treatment of the relativity theory differs essentially from those discussed above, for while their approach is from the standpoint of physics, this book develops the mathematical side of the theory with comparatively little attention to physics. The reader's knowledge of the calculus is presupposed, but nothing further. The author states in his Preface that he wishes the book to be regarded as "written by a layman for other laymen who are, so to speak, a few lessons behind him." With this encouraging beginning he plunges into the task of removing difficulties. Chief of these is the postponement of the unavoidable tensor calculus to the last chapters, where its unfavorable effect on readers is minimized. Thus Dr. Nunn is required to treat gravitational theory, and

in particular the motion of a planet or light-ray in the field of the sun, without the aid of tensor theory. He must omit Einstein's field equations of gravitation, and set up directly their solution in the required case. Although this cannot be accomplished without introducing unjustified hypotheses, as may be demonstrated by an example, it is the only method of reaching the goal in an entirely elementary way, and its logical imperfections are thus inherent in the mathematical restrictions imposed.

H. C. LEVINSON

The Electrical Conductivity of the Atmosphere and Its Causes. By VICTOR F. HESS; trans. from the German by L. W. CODD. London: Constable & Co., 1928. Pp. xviii+204. 12*s*.

The experiments we used to perform with electrified brass spheres, ellipsoids, ovals, cylinders, and what not, mounted on glass stems, always were beset with one difficulty: the charge would disappear. In damp weather it was impossible to get any evidence of a charge at all because the film of moisture on the support conducted it away as soon as supplied. Even when the support was an almost perfect insulator, the electrification slowly vanished. We said that the conductor lost its charge by progressively sharing it with the adjacent air and then driving that air away; and we proved this theory too by experiments—we even blew out lighted candles with the electric wind from sharp points! This seemed conclusive, and was until someone found that an insulated conductor lost an electric charge faster in dry air than in humid air.

That set experiments going again, and then faster and faster a multitude of new things were found. The conductor does not share its electrification with the adjacent air. The atmosphere everywhere is full of little charges, both positive and negative, 600–800, generally, of each per cubic centimeter, and the conductor becomes discharged by drawing unto itself charges of the opposite sign out of the atmosphere. The magnitude of the individual charge in the air has been measured. It usually is that of the single electron. The mobility of the carrier also has been determined. Its average value is much greater in country air than in that of cities or about industrial plants. In pure air the ions are relatively small. In smoky air they are comparatively very massive. This means that the atmosphere has an electrical conductivity, just as an electrolyte has, and for the same reason—the presence of ions.

We know now that the ionic content of the air varies under different conditions and especially with height. Several radically different sources of the ions have been found, and two or three ways by which they cease to be ions. We know that at 100 km or so above the surface of the earth

the atmosphere is greatly ionized—this makes wireless communication possible. And there are scores of other things about atmospheric electricity we now know that no one ever imagined just a little while ago.

The story of atmospheric electricity is a detective story in the realm of nature, arresting in the wealth of material already in hand, and alluring through the promise of rich finds just ahead. This story is told in the accurate language of the scientist in the foregoing work by one of the chief investigators in this important branch of physics.

W. J. HUMPHREYS

Graphische Darstellung der Spektren von Atomen und Ionen mit Ein, Zwei und Drei Valenzelektronen. By W. GROTRIAN. "Struktur der Materie," Band VII. Berlin: Julius Springer, 1928. Vol. I, pp. xiii+245, Figs. 43; Vol. II, pp. x+168, Figs. 163. Unbound, 34.00 RM; bound, 36.40 RM.

The author of this book has made a valuable correlation of the widely scattered data pertaining to the spectra of atoms and ions having one-, two-, and three-valence electrons. The second volume will be particularly useful to spectroscopists and astrophysicists, since it consists entirely of diagrams which show at a glance the structure of all the simpler spectra. An original feature of these diagrams, whose general form was first proposed by the author several years ago, is the separate representation of sublevels in multiple spectroscopic terms. This makes it possible to show the individual spectral lines in every multiplet. The wave-length of each line is given, and its relative intensity is indicated by the blackness of the printed line. There is a set of diagrams showing the relative levels of terms in homologous spectra, a group of Moseley diagrams and charts for the irregular and regular doublet laws. Finally, there are diagrams for the "anomalous" terms and their combinations with the "normal" terms in spectra having two electrons. There is much to be said for the grouping together of terms arising from the same electronic configuration in these diagrams.

The first volume describes in detail all of the spectra which come within its scope, and includes a chapter on X-ray spectra and another on the Moseley law and on the irregular and regular doublet laws. The author does not attempt to bring the theoretical side of the subject up to date since this has been fully covered by Hund in his *Linienspektren und periodisches System der Elemente*, an earlier volume in the present series. We feel, however, that a somewhat more complete discussion of the Hund theory and of the Pauli restriction would have made easier the

author's task of explaining the observed terms in spectra on the basis of electronic structure.

The spectroscopic notation is not uniform throughout the book, and this leads to a slight confusion at times. Paschen's notation is used to a large extent, but in many places the author also employs a modification of the Russell and Saunders notation with fractional inner-quantum numbers for the doublets. This has theoretical advantages, although the symbols are necessarily a little more difficult to read. The convention with respect to the primes in the Russell and Saunders notation is not brought out clearly, and no mention is made of the classification of spectroscopic terms as "even" or "odd" depending on the combined number of p- and f-electrons. In two- and three-electron spectra the author distinguishes terms having the same value of L by giving the complete electronic configuration, e.g., $3s3p3p^2P_{\frac{1}{2}}$. Bowen's abbreviated notation for the common configurations is probably the best solution of the problem that has so far been proposed, although for descriptive purposes the extended form is entirely justified.

The diagrams include a scale of excitation potentials in volts running upward from the lowest level in the atom, but the term-values are in every case measured downward from the lowest series-limit. In view of the fact that all but the simplest spectra have several series-limits, there is at present an increasing tendency to count term-values upward from the lowest energy-level. This has the great incidental advantage that the term-values become directly proportional to excitation potentials. For complex spectra, where series-limits are in general only approximately known from the application of a Rydberg formula to the first two terms, we have no choice in the matter, unless we are prepared to revise the levels of all the terms whenever an improved value of the series-limit becomes available.

The only serious misprint which we have detected is in Figure 85, "II" for "*Si* II." The lines at λλ 3853, 3856, and 3862 are shown as combinations between the terms $2p$ and x, whereas they are actually combinations between x and $3p$. The transition $2p$-x gives the lines in the ultra-violet at λλ 1808, 1817, and 1817. Bowen[1] has shown that the x-term is a ^2D-term belonging to the sp^2-configuration (see **1**, 218).

The usefulness of the book would be considerably increased if it contained tables giving the numerical values of the terms for the spectra considered. But it fills a real need, and the second volume especially with its valuable diagrams, will be frequently consulted.

THEODORE DUNHAM JR.

[1] *Physical Review*, **31**, 34, 1929.